On the non-neutrality and optimality of monetary policy when financial markets are incomplete: a macroeconomic perspective

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Summary
We study in this paper a simple model of a two-period economy, with two states of the world in the second period, two agents and one good. Financial markets are incomplete since only inside money is available. We show that outside money, which is introduced in the model through its role as a medium of exchange, is non-neutral, in the sense that it has an effect on the equilibrium allocation. We then discuss whether a monetary policy that would aim at state-independent price levels is desirable. We illustrate that discussion with a few examples. The possible sub-optimality of a constant-across-states inflation rates target for monetary policy is to be contrasted with results from representative agent macroeconomic models.

J.E.L. Classification: D52, E52.
Keywords: Monetary economics, incomplete market, general equilibrium.

1. Introduction
The objective of this paper is to study the role of monetary policy in a simple general equilibrium model with incomplete markets.

Two main transmission channels of monetary policy to real variables are usually distinguished in macroeconomic models; the first is based on the inflationary tax on intertemporal money holdings, while the second relies on the real balance effect, as in models with nominal rigidities.

Distribution effects are usually not taken into account in this standard representative agent framework, although they are an
important part of the transmission of monetary policy to real variables, especially when contracts are stated in nominal terms.†

When assets are nominal,‡ variations of the price level across states of the world lead to changes in the purchasing power of assets' returns. If markets are incomplete—i.e. if there are less independent assets than states of the world—and when there is no outside money, the purchasing power of a unit of account is not endogenously determined. This is what drives the indeterminacy of the equilibrium typical in these models (Balasko & Cass, 1989; Geanakoplos & Mas-Colell, 1989).

Magill and Quinzii (1992) point out that the indeterminacy property becomes one of non-neutrality of monetary policy if outside money is introduced in the model through its role as a medium of exchange. Introducing outside money then allows one to determine endogenously the price level in each state. In a model where money serves both as a medium of exchange and as a store of value, local uniqueness of the equilibrium is restored, but the equilibrium allocation is now parameterized by a monetary policy variable. The income distribution reached is therefore dependent on monetary policy and the model could then produce an income distribution that would be unreachable, were prices constant across states.

We choose to illustrate this property in a simple two-period pure exchange economy with two states of the world, one good, two agents and inside money. The simple approach we follow enables us to explicitly parameterize the equilibrium by a money supply indicator.

We show that aiming for a constant rate of inflation across states (i.e. equal prices in different states) is not necessarily optimal for the economy. In other words, the "optimal price level" could well be different in each state. Intuitively, if agents can not fully insure themselves against the second-period risk, a monetary policy that would create large distortions among the prices in different states could allow for a better agents' coverage. We provide a partial characterization of economies where aiming at constant-across-states inflation rates is not desirable. We show through examples where agents have different endowments and preferences in the different states, that there is no simple monetary rule that would always be optimal, irrespective of the characteristics of the economy under consideration.

Neumeyer (1992) studies efficiency issues in an incomplete market model where money is only a unit of account. He shows that large variability of the rate of inflation reduces the utility possibility

† See Allais (1993) for a discussion on this point.
‡ That is to say, assets whose returns are denominated in units of account, as opposed to real assets whose returns are commodities.
set through a contraction of financial markets. Cass et al. (1992) and Gottardi (1994) introduce outside money as a store of value in an overlapping generation model with incomplete markets, and show in this framework that monetary policy matters. Yet another way of introducing money in such models is to assume that agents must pay taxes with outside money, as in Villanacci (1991, 1992).

After describing the model in the first section, we show in Section 3 how monetary policy matters when markets are incomplete. Optimality issues are discussed in Section 4. A few examples illustrating that point are gathered in Section 5.

2. The model

We deliberately place ourselves in the simplest pure exchange economy framework† to illustrate the role of monetary policy when financial markets are incomplete.

2.1. NOTATION AND PREFERENCES

The first period is denoted 0 while the two second-period states are α and β. Agents exchange the good at each spot and can trade an asset in the first period. This asset pays one unit of account in the second period, no matter what the state is. It will be referred to as inside money. Consumption of the good in spot s = 0, α, β by agent h = 1, 2 is denoted x_h(s); the good’s price in state s is p(s).

The vector of agent h’s consumption will be denoted x_h = (x_h(0), x_h(α), x_h(β))’, and p = (p(0), p(α), p(β)) is the price vector. Asset holdings are denoted b_h, h = 1, 2 and its price is q. Agents’ initial endowments are equal to e_h = (e_h(0), e_h(α), e_h(β))’ and their preferences are represented by utility functions u_h from \( \mathbb{R}_+^3 \) to \( \mathbb{R} \).

We make the following standard assumptions:

Assumption 1: (i) \( u_h \) is \( C^2 \), strictly increasing \( (Du_h(x_h) \gg 0) \), differentiably strictly quasi-concave—i.e. \( z \neq 0 \) and \( Du_h(x_h)z = 0 \Rightarrow z'D^2u_h(x_h)z < 0 \). Its indifference curves are closed in \( \mathbb{R}_+^3 \). (ii) \( e_h \in \mathbb{R}_+^3 \).

2.2. OUTSIDE MONEY AND MARKET ORGANIZATION

We now introduce outside money in the model, in order to study the role of monetary policy when financial markets are incomplete. We follow Magill and Quinzii (1992) and assume that a central

† We also deal with production in Portier and Tallon (1993).
institution (say a government) issues outside money at each spot, and that agents hold it for transaction purposes. Why would this exogenously fixed amount of money be valued by private agents? We adopt a simple institutional story that gives money a role as a medium of exchange and a unit of account.

During the first period, agents sell their initial endowments to the government and receive money in exchange, denoted $m_h(0)$. This amount of money is then used to buy goods from the government as well as inside money from other agents.

The exchange process is basically the same in the second period: agents sell their initial endowments for outside money, $m_h(s)$, $s = \alpha, \beta$. This money is then used to buy the good and pay the asset's return. It could be noticed that this way of introducing outside money entails that a quantitative equation holds spot by spot. The price level adjusts so that the total value of wealth is equal to the amount of money printed by the government.

The budget set of household $h$ is thus defined by the following constraints that describe the selling of initial endowment and the possible use of the money she gets from it:

\[
\begin{align*}
    p(0)e_h(0) &= m_h(0) \\
    m_h(0) &\geq p(0)x_h(0) + q_h \\
    p(\alpha)e_h(\alpha) &= m_h(\alpha) \\
    m_h(\alpha) &\geq p(\alpha)x_h(\alpha) - b_h \\
    p(\beta)e_h(\beta) &= m_h(\beta) \\
    m_h(\beta) &\geq p(\beta)x_h(\beta) - b_h.
\end{align*}
\]

Since outside money has no utility and cannot be held from one period to the other, the inequalities in the system above can be written as equalities without loss of generality. The maximization program agent $h$ has to solve is then given by ($P_h$):

\[
\max_{x_h(0), x_h(\alpha), x_h(\beta)} u_h(x_h)
\]

† Contrary to Magill and Quinzii (1992), we assume that agents cannot hold outside money intertemporally. Thus, we emphasize the role of money as a medium of exchange rather than its role as a store of value. Given the effect we want to analyse—i.e. that there exists monetary policies that have a real impact without having to assume the existence of an inflationary tax—this assumption is not too restrictive. Indeed, the transmission channel of monetary policy towards real variables that we analyse relies upon the use of outside money in the exchange process and the incompleteness of financial markets. In that view, the assumption that agents cannot hold money from one period to the other is not crucial.
It is easy to see that the three constraints $m_h(s) = p(s)e_h(s)$ are of no real importance in the maximization problem and only define the amount of outside money $m_h(s)$ agent $h$ gets. In particular, as already pointed out, households use only inside money as a store of value.

2.3. COMPETITIVE FINANCIAL EQUILIBRIUM

Denoting by $M(s), s = 0, \alpha, \beta$ the aggregate supply of outside money in state $s$, we can define a competitive financial equilibrium in this model.

DEFINITION 1: the vector $(p, q, x_1, b_1, m_1, x_2, b_2, m_2)$ is a competitive financial equilibrium of the economy if:

(i) $(x_1, b_1)$ is a solution to $(P_h)$ for $p$ and $q$;
(ii) $\sum_h x_h(s) = \sum_h e_h(s), s = 0, \alpha, \beta$, and $\sum_h b_h = 0$;
(iii) $M(s) = \sum_h m_h(s), s = 0, \alpha, \beta$.

3. Monetary policy and financial equilibrium

We show in this section that monetary policy is non-neutral in the sense that the equilibrium goods allocation depends on it. Even though we could have applied Magill and Quinzii's (1992) result (our economy being a particular case of their model), we chose to give a proof, due to Cass (1985), that is both intuitive and rigorous, of the parameterization of equilibrium allocations by a monetary policy variable in our framework. In other words, we define a parameter, that the government controls through its monetary policy, which can be used as an index for the equilibrium allocations.

† Magill and Quinzii (1992) show that generically in initial endowments, a non-proportional change of the monetary variables $M(s), s = 1, \ldots, S$ is non-neutral.
3.1. REDUCING H’S MAXIMIZATION PROGRAM

Our proof can be facilitated by rewriting the problem \((P_h)\).

In a first step, we omit the equations \(m_h(s) - p(s)e_h(s), s = 0, \alpha, \beta\) in the program \((P_h)\), since they represent identities defining outside money holdings. This operation does not change agent \(h\)'s budget set. The second step consists in expressing \(x_h(\beta)\) as a function of \(x_h(\alpha)\) using the budget constraints in state \(\alpha\) and \(\beta\). Finally, it is possible to replace \(b_h\) in the first period budget constraint by its value computed from the budget constraint in state \(\alpha\). These transformations yield the following program \((P'_h)\) equivalent to \((P_h)\):

\[
\max_{x_h(0), x_h(\alpha), x_h(\beta)} u_h(x_h(0), x_h(\alpha), x_h(\beta))
\]

\[
\begin{cases}
  p(0)(x_h(0) - e_h(0)) + q p(\alpha)(x_h(\alpha) - e_h(\alpha)) = 0 \\
  p(\alpha)(x_h(\alpha) - e_h(\alpha)) = b_h \\
  \frac{p(\alpha)}{p(\beta)} (x_h(\alpha) - e_h(\alpha)) + e_h(\beta) = x_h(\beta)
\end{cases}
\quad \text{s.t.} \quad x_h \in \mathbb{R}_{++}^3.
\]

Written this way, it appears that the second constraint is an identity defining the quantity of asset \(b_h\) held by \(h\), and that the third constraint defines \(x_h(\beta)\). Let \(\theta = p(\alpha)/p(\beta),\) the program \((P'_h)\) can be transformed into the program \((P''_h)\):

\[
\max_{x_h(0), x_h(\alpha)} u''_h(x_h(0), x_h(\alpha))
\]

\[
\begin{cases}
  p(0)(x_h(0) - e_h(0)) + q p(\alpha)(x_h(\alpha) - e_h(\alpha)) = 0 \\
  (x_h(0), x_h(\alpha), \theta(x_h(\alpha) - e_h(\alpha)) + e_h(\beta)) \in \mathbb{R}_{++}^3
\end{cases}
\quad \text{s.t.} \quad (x_h(0), x_h(\alpha), \theta(x_h(\alpha) - e_h(\alpha)) + e_h(\beta)) \in \mathbb{R}_{++}^3
\]

where:

\[
u''_h(x_h(0), x_h(\alpha)) = u_h(x_h(0), x_h(\alpha), \theta(x_h(\alpha) - e_h(\alpha)) + e_h(\beta)).\]

The agent's problem is hence reduced to a standard intertemporal maximization problem, with only one state tomorrow and complete financial markets. We denote that program by \((P''_h)\). Note that in this new specification a shift in \(\theta\) acts like a shift in agents' preferences.

It is easy to show that \(u''_h\) satisfies assumption 1(i) when \(u_h\) does. The rewritten model is then a standard general equilibrium model.

\[\footnote{This parameter, which can be written, for equilibrium prices, as \((M(\alpha)/M(\beta))(\sum_i e_h(\beta)/\sum_i e_h(\alpha))\), will turn out to be the monetary policy instrument.}\]
for which we know, given our assumptions, that there exists an equilibrium defined in the following way:

**Definition 2**: the vector \((p'(0), p'(\alpha), x'_0(0), x'_0(\alpha), x'_\alpha(0), x'_\alpha(\alpha))\) is a competitive equilibrium of the reduced economy if:

(i) \((x'_h(0), x'_h(\alpha))\) is a solution to \((P_e')\) at prices \(p'(0)\) and \(p'(\alpha)\),

(ii) \(\sum_s x'_h(s) = \sum_h e_h(s), s = 0, \alpha.\)

Observe that implicit in this definition is the fact that the asset price, \(q'\), is normalized to 1. This entails no loss of generality as the budget constraint in \((P_e')\) is linear homogeneous in \(p(0)\) and \(qp(\alpha)\).

### 3.2. The Real Effect of Monetary Policy

Theorem 1 states that the existence of an equilibrium in the reduced economy implies the existence of a financial equilibrium parameterized by \(\theta\), in the original economy.

**Theorem 1**: for \(\theta\) given by \(\theta = (M(\alpha)M(\beta))(\sum_h e_h(\beta)\sum_h e_h(\alpha))\), there exists a competitive financial equilibrium \((p, q, x, b, m)\) of the original economy.

**Proof**: see the Appendix.

We now show that when the parameter \(\theta\) changes—i.e. when the government is introducing different amounts of outside money in states \(\alpha\) and \(\beta\)—the equilibrium allocation changes. Thus, a monetary policy change does not affect only prices, as it would have been the case in a representative agent version of our model, but also implies a change in the quantities exchanged and consumed by the agents. Observe also that it is perfectly anticipated. The role of monetary policy is made precise in theorem 2, which rests upon the construction of the equilibrium in the original economy from the one in the reduced economy as explained in the proof of theorem 1.

**Theorem 2**: if the initial endowment \(e\) is not a Pareto optimal allocation, the equilibrium allocation for the policy \(\theta\) is different from the one for the policy \(\theta \neq \theta\).

**Proof**: see the Appendix.

The proof relies on showing the following two propositions: (i) There is at most one \(\theta\) such that the equilibrium allocation of the
reduced economy is equal to the initial endowments, (ii) two different policies \( \bar{\theta} \) and \( \bar{\theta} \) lead to different equilibrium allocations \( \bar{x} \) and \( \tilde{x} \).

It is important to stress the fact that the reason for the non-neutrality of money in this framework is different from the traditional effects in macroeconomic models. The effect under consideration here rests on the impossibility for agents to freely allocate their wealth across states of the world. It is then clear that a policy that changes the inflation differential between states, against which agents cannot hedge, will be non-neutral even if it is perfectly anticipated.

The impact of monetary policy relies crucially on agents' heterogeneity. If agents were identical, exchanges on asset markets would be identically equal to zero and monetary policy would then be neutral. This is also the case if markets are complete, as we now proceed to show.

3.3. THE NEUTRALITY OF MONETARY POLICY WHEN MARKETS ARE COMPLETE

We modify the model by introducing a second asset, whose payoffs are linearly independent of those of inside money. Without loss of generality, we will assume that that second asset pays one unit of account in state \( \beta \) and zero in \( \alpha \).

Agent \( h \)'s problem \((P_h)\) when markets are complete can be written in the following way:

\[
\max_{x_h, b_h} u_h(x_h)
\]

\[
\begin{align*}
p(0)(x_h(0) - e_h(0)) &= -q_1 b^*_h - q_2 b^2_h \\
p(\alpha)(x_h(\alpha) - e_h(\alpha)) &= b^*_h \\
p(\beta)(x_h(\beta) - e_h(\beta)) &= b^*_h + b^2_h \\
m_h(0) &= p(0)e_h(0) \\
m_h(\alpha) &= p(\alpha)e_h(\alpha) \\
m_h(\beta) &= p(\beta)e_h(\beta) \\
x_h &\in \mathbb{R}^3_{++}.
\end{align*}
\]

The definition of a financial equilibrium remains the same, and we easily get the neutrality result.

**Theorem 3:** the competitive financial equilibrium of the complete market economy is independent of \((M(0), M(\alpha), M(\beta))\).

**Proof:** see the Appendix.
Thus, when markets are complete, any change in the monetary policy will be entirely absorbed through nominal changes, i.e. changes in the price levels. This is not surprising since the reason why monetary policy was relevant in the incomplete market economy, is the impossibility for agents to fully insure themselves against the second period risk. In other words, they can now allocate revenue state by state, whereas they could only transfer a global amount from one period to the other when markets were incomplete.

4. Should inflation be independent of the state of the world?

We have shown in the previous section that money matters when markets are incomplete. The next natural question to ask is what is the optimal policy, if there is one. We'll say that a monetary policy is optimal if there is no other monetary policy whose associated allocation is Pareto superior. Should the monetary authority adopt a rule whose target is a particular, state independent, rate of inflation, or should it follow a discretionary policy that would allow for price level differences across states? The issue at stake is important, even if one should keep in mind that we look at it in an over-simplified setup. Should the monetary authority accommodate a real shock, or should it adopt a rule independent of each state’s fundamentals? More concretely, should the monetary authority announce that its policy will be tight, regardless of whether there is say, an oil shock or not?

The answer to these questions are of course dependent on agents’ characteristics. More formally, for given preferences, let \( u_h(p(0), p(x), \theta; e_h) \) be the indirect utility of agent \( h \), at the equilibrium indexed by \( \theta \). The utility functions having all the right properties, demand functions are differentiable, and thus the indirect utility functions are differentiable as well. If the partial derivative with respect to \( \theta \), of both indirect utility functions are non-zero and have the same sign at \( \theta = 1 \), constant-across-states price levels is clearly not the right target for monetary policy. Indeed, the monetary authority can improve both agents’ welfare by changing \( \theta \), i.e. by allowing for different price levels in the two states.

Now, if those partial derivatives have opposite signs, there is no local change of \( \theta \) that would benefit both agents. In that sense, aiming at state-independent prices is not locally dominated by another policy. However, another policy might exist that gives both agents higher utility levels than when \( \theta = 1 \).

Is it possible to find conditions on endowments and preferences that would ensure that this condition on derivatives of indirect utilities is met? Observe first that, for given preferences, if the above derivatives are of the same sign for some \( e \), then this property
still holds for endowments in an open neighbourhood of \( e \). Similarly, if the derivatives are of opposite signs for some endowments \( e \), it is still the case in a neighbourhood of \( e \). Therefore, since the two cases are plausible, there is not much hope to find generic sets of economies such that one particular policy is always desirable.

Nevertheless, there is an (admittedly rather particular) class of models where aiming at a constant-across-states price level is not the right policy. Indeed, observe that if the proposition “if \( \theta = 1 \rightarrow b_h = 0, h = 1, 2 \)” is true, then any policy is at least as good as the one aiming at constant-across-states price levels. This is the case for the obvious reason that when \( b_h = 0 \) the endowment is the equilibrium allocation. This allocation is also reachable if \( \theta \neq 1 \). Therefore, if the equilibrium allocation is different from the endowments for \( \theta \neq 1 \), the former necessarily Pareto dominates the latter. In other words, if \( \theta = 1 \) leads to a no-trade equilibrium, constant price levels is the worst possible policy.

This class of economies can easily be characterized by the equality of agents’ marginal rate of substitutions evaluated at the endowment point in the reduced economy when \( \theta = 1 \). Formally, this yields:

\[
\frac{D_{x(0)}u_1(e_1)}{D_{x(0)}u_1(e_1) + D_{x(0)}u_2(e_1)} = \frac{D_{x(0)}u_2(e_2)}{D_{x(0)}u_2(e_2) + D_{x(0)}u_2(e_2)}.
\]

This is met, for instance, for economies where agents have identical preferences, and where the risk is purely a microeconomic risk, that is \( e_1(0) = e_2(0), e_1(\alpha) = e_2(\beta) \) and \( e_1(\beta) = e_2(\alpha) \), with \( e_1(\alpha) \neq e_1(\beta) \).

5. An illustration of the effectiveness of monetary policy

A few examples are exposed in this section, which enable one to see that the effects of monetary policy can be complex even in simple cases.

We adopt a log-linear specification of the utility function with \( z_h \) denoting the coefficient of the good in state \( s \) for agent \( h \)

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† If agents are identical, i.e. in the case of a representative individual, it is the best policy as well since \( \theta \) has no impact on the equilibrium allocation.

‡ One can specify a probabilistic structure for the second period uncertainty and assume Von Neuman–Morgenstern preferences without altering the qualitative results of the model.
An economy is hence characterized by a vector \((z^s_h)\), \(h=1, 2\) and \(s=0, \alpha, \beta\) describing agents' preferences and initial endowments \((e^s_h)\), \(h=1, 2\) and \(s=0, \alpha, \beta\).

In the set of economies under consideration, we adopt the normalization \(M(0)=M(\alpha)=10\). We then plot the utility levels \(u_1\) and \(u_2\) corresponding to equilibrium allocations when the policy parameter \(\theta\) varies, as well as the solution (independent of \(\theta\)) when markets are complete.

5.1. EXAMPLE 1

Agents have the same preferences. Their endowments in the second period are constant across the states. There is therefore no risk on the fundamentals of the economy. With identical money supply in the two states, price levels are the same \((\theta = 1)\) and the distinction between state \(\alpha\) and state \(\beta\) is irrelevant. Agents thus face complete financial markets, and the economy reduces to a two-period model without any uncertainty. For \(\theta = 1\), the equilibrium is an optimum; see Table 1 and Figure A.
What are the effects of a state-dependent monetary policy in this example? When money supply is different in the two states, price levels are themselves distinct thus creating a macroeconomic risk against which agents cannot insure, since only nominal assets are available. As seen in Figure A, such a situation lowers their utility. Hence, any policy such that $\theta \neq 1$ yields allocations that are Pareto inferior to the ones with constant-across-states monetary policy.

In this particular economy with no risk on the fundamentals, price level discrepancies make agents worse off.

One could have noticed that the economy under study is of the sunspot type, à la Cass (1989). However, the conclusion we drew cannot be extended to a general class of sunspot economies. It is indeed possible to construct examples where the equilibria associated to $\theta = 1$ and $\theta \neq 1$ cannot be Pareto-ranked.†

5.2. Example 2

Agents have identical preferences, identical endowments in the first period and symmetrical second-period endowments. All the risk on the fundamentals of the economy are at the individual level. There is hence no macroeconomic risk other than the one introduced through monetary policy. With complete financial markets, individual risk is perfectly pooled, each agent consuming the same amount in the two states. However, when markets are incomplete and monetary policy is constant across states, there is no exchange and each individual bears its own risk. An active monetary policy, through its impact on price levels, creates a gap between state $\alpha$ and state $\beta$ real rate of return of the asset. It therefore allows for partial insurance among agents, through mutually beneficial exchanges on the asset market. (See Table 2 and Figure B.)

This example is the opposite to the previous one, and is representative of a class of economies where a differential of inflation

† We thank a referee for pointing this out.
between states is always Pareto improving with respect to the constant-across-states monetary policy. Indeed, the sufficient condition we exhibited in the previous section is satisfied here. Finally, one could observe in this example that there is a continuum of Pareto-ranked equilibria indexed by $\theta$, and therefore that there is strictly speaking no optimal monetary policy.

5.3. **Example 3**

It is shown in this example that monetary policy has no systematic effect in the class of economies under consideration.

Here, agents differ in endowments and preferences. Monetary policy is not always Pareto improving. However, there are levels of $\theta$ for which both agents essentially achieve their complete market utility level. An agent can also have a utility level when markets are incomplete strictly greater than in the complete markets case, for some values of the monetary policy parameter. (See Table 3 and Figure C.)
6. Conclusion

We gave in this paper a simple intuition of how the monetary policy debate can be fruitfully undertaken in a world with heterogeneous agents. We then illustrated the effectiveness of monetary policy when markets are incomplete. These effects are based on distribution issues between creditors and debtors.

It is clear that these examples are too particular to be of great empirical relevance. It is nevertheless worth pointing out that we found a class of economies, where the risk is entirely microeconomic, in which a differential of inflation among states is a good policy. The interest of the other examples are then to show that the issue of optimal monetary policy cannot be discussed in full generality without referring to a specific economy. One can see that even in very simple cases, there is no unambiguous answer to the question “what is the optimal monetary policy?”, as soon as markets are supposed to be incomplete.

Acknowledgements

We thank participants at the M.A.D. seminar, Helsinki E.E.A. '93 meeting, as well as two anonymous referees for useful comments. Discussions with J. P. Bénassy were also very helpful.

References

Appendix

PROOF OF THEOREM 1: an equilibrium of the reduced economy exists. Indeed, it is easy to check that standard existence conditions are fulfilled (in particular, the price dependent consumption set is closed, convex and bounded below).

Observe first that if \((p'(0), p'(a), x'_0(0), x'_0(a), x'_0(0), x'_0(a))\) is an equilibrium of the reduced economy, then for \(a \in R_{++}\),

\[(ap'(0), ap'(a), x'_0(0), x'_0(a), x'_0(0), x'_0(a)),\]

is an equilibrium as well.

One can then normalize \(p'(0)\) in the following way:

\[p'(0) \equiv \frac{M(0)}{\sum_h e_h(0)},\]

and define

\[p(0)=p'(0), p(a)=\frac{M(a)}{\sum_h e_h(0)}, q=p'(a) p(a),\]

where \(p'(a)\) is such that \((p'(0), p'(a))\) is an equilibrium of the reduced economy for \(\theta=(M(a)/M(\beta)) (\sum_h e_h(\beta))/\sum_h e_h(\alpha))\). Using the definitions
We get

\[ x_h(\beta) = \frac{M(x) \sum_h e_h(\beta)}{M(\beta) \sum_h e_h(x)} (x_h(x) - e_h(x)) + e_h(\beta) \]

\[ b_h = p(x)(x_h(x) - e_h(x)) \]

\[ m_h(s) = p(s)e_h(s), \quad s = 0, x, \beta. \]

The constraint on \( \theta(x_h(x) - e_h(x)) + e_h(\beta) \) ensures that \( x_h(\beta) \) is positive.

It is easy to show that:

\[ \sum_h (x_h(x) - e_h(x)) = 0 \Rightarrow \begin{cases} \sum_h (x_h(\beta) - e_h(\beta)) = 0, \\ \sum_h b_h = 0. \end{cases} \]

Finally, we have

\[ p(\beta) = \frac{p(x)}{\theta} = \frac{M(x)}{M(\beta)} \frac{\sum_h e_h(x)}{\sum_h e_h(\beta)} = \frac{M(x)}{\sum_h e_h(\beta)} \]

and it is then easy to check the conditions:

\[ \sum_h m_h(0) = p(0) \sum_h e_h(0) = M(0), \]

\[ \sum_h m_h(x) = p(x) \sum_h e_h(x) = M(x), \]

\[ \sum_h m_h(\beta) = p(\beta) \sum_h e_h(\beta) = M(\beta). \]

The vector \((p, q, x, b, m)\) thus defined is hence a financial equilibrium of the economy.

\[ \square \]

**PROOF OF THEOREM 2**: we prove the following two propositions: (i) There is at most one \( \theta \) such that the equilibrium allocation of the reduced economy is equal to the initial endowments, (ii) two different policies \( \theta \) and \( \tilde{\theta} \) lead to different equilibrium allocations \( \bar{x} \) and \( \tilde{x} \).

(i) A necessary condition for \((e_h(0), e_h(x))\) to be an equilibrium of the reduced economy is that there is \( \lambda \) such that

\[ (D_{x_h(0)} u_1(e_1), D_{x_h(x)} u_1(e_1) + \theta D_{x_h(\beta)} u_1(e_1)) \]

\[ = \lambda (D_{x_h(0)} u_2(e_2), D_{x_h(x)} u_2(e_2) + \theta D_{x_h(\beta)} u_2(e_2)). \]
Since \( e \) is not, by assumption, a Pareto optimum (that is to say, \( D_xu_1(e_1) \) and \( D_xu_2(e_2) \) are not colinear), this equality can hold for at most one \( \theta^* \).

(ii) Let \( \theta \neq \tilde{\theta} \) and \( \tilde{x} \) and \( \tilde{\tilde{x}} \) be the associated equilibrium allocations in the respective original economies.

If \( \tilde{x}(x) \neq \tilde{\tilde{x}}(x) \), then proposition (ii) is trivially true and different monetary policies yield different equilibrium allocations. If, in turn, \( \tilde{x}(x) = \tilde{\tilde{x}}(x) \), then \( \tilde{x}(x) \neq e_h(x) \) or \( \tilde{\tilde{x}}(x) \neq e_h(x) \), since otherwise \( e \) would be an equilibrium in the reduced economy for \( \theta \) and \( \tilde{\theta} \), which is contradicting (i). Hence, we have:

\[
\tilde{x}_h(x) - e_h(x) = \tilde{\tilde{x}}_h(x) - e_h(x) \neq 0,
\]

which implies that \( \tilde{x}_h(\beta) \neq \tilde{\tilde{x}}_h(\beta) \). This proves the result. \( \square \)

**Proof of Theorem 3:** one gets from the constraints in \((P_h^\alpha)\), \( h = 1, 2 \), the equilibrium relation:

\[
\frac{M(s)}{\sum_h e_h(s)}, \quad s = 0, x, \beta.
\]

The assets' prices \( q^1 \) and \( q^2 \) satisfy then:

\[
\begin{cases}
q^1 p(x) - q^2 p(x) = p'(\alpha) \\
q^2 p(\beta) = p'(\beta)
\end{cases}
\]

where \((M(0)/(\sum_h e_h(0)), p'(\alpha), p'(\beta))\) is the equilibrium of the reduced economy, in which household \( h \) solves the following program \((P_h^\alpha)\):

\[
\max_{x_h} u_h(x_h)
\]

s.t. \( p(0)(x_h(0) - e_h(0)) + (q^1 - q^2)p(\alpha)(x_h(\alpha) - e_h(\alpha)) + q^2 p(\beta)(x_h(\beta) - e_h(\beta)) = 0 \) \((P_h^\alpha)\)

with the first period normalization \( p(0) = M(0)/(\sum_h e_h(0)) \).

The equilibrium allocation in the reduced economy is also an equilibrium of the complete market economy. This allocation is independent of the monetary policy. \( \square \)