Disclaimer

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0. Introduction

What is this lecture about?

- Existing concerns regarding the earth’s population explosion and the pressure it places on natural resources
- Debate on *sustainable growth* in the light of a possible global warming of the earth.
- In this Lecture, we will first discuss of an example of an economic, social and demographic collapse (Easter Island) ([Jarred Diamond](#)[2005])
- We will then present a dynamic model with endogenous population and a model for the dynamics of the ecosystem.
- We will then be able to discuss the possibility of an “Easter Island” type of collapse, and what are the important economic and biological forces in such a dynamics.
0. Introduction

Plan of the Lecture

1. A Short Story of Easter Island
2. A Model for the dynamics of Easter Island
3. Applying the Model to Easter Island
4. Summary
5. References
1. A Short Story of Easter Island

Geography

- Easter Island (also called Rapa Nui) is a small Pacific island, 3200km from the coast of Chile and 7000km from New-Zealand), with a population (as of the early 1990’s) of about 2,100.
- Easter is a triangular island consisting entirely of three volcanoes that arose from the sea.
- Subtropical location at latitude 27 degrees south, approximately as far south of the equator as Miami and Taipei lie north of the equator
1. A Short Story of Easter Island

Geography

Figure 1: Location of Easter Island
1. A Short Story of Easter Island

Geography

Figure 2: Location of Easter Island
1. A Short Story of Easter Island

Geography

- Mild climate
- Fertile soils because of its recent volcanic origins
- While a subtropical climate is warm by the standards of European and North American winters, it is cool by the standards of mostly tropical Polynesia
- Modest rainfalls.
- The rain that does fall percolates quickly into Easter’s porous volcanic soils. As a consequence, freshwater supplies are limited.
- Easter is a windy place
- Easter’s isolation meant, that it is deficient not just in coral-reef fish but in fish generally, of which it has only 127 species compared to more than a thousand fish species on Fiji.
1. A Short Story of Easter Island

Geography

Figure 3: Landscape
1. A Short Story of Easter Island

Geography

Figure 4: Volcanos
1. A Short Story of Easter Island
Discovery by Europeans

- First European contact in 1722 (by three Dutch ships). This visit lasted only a single day.
- The next known contact was a brief visit from a Spanish ship in 1770.
- In 1774, visit from James Cook, who provided a systematic description of Easter Island.
- A desertic island with about 3,000 inhabitants.
1. A Short Story of Easter Island

The Moai

- Moai: enormous statues carved from volcanic stone.
- 887 on the island.
- Many statues rested on large platforms (ahu) made of rubble.
- About 300 ahu have been identified, of which many were small and lacked moai, but about 113 did bear moai.
- The largest ‘movable’ statues weigh more than 80 tons, far form the quarry.
- The largest statue of all lies unfinished in the quarry where it was carved, and weighs about 270 tons.
1. A Short Story of Easter Island

Moai

Figure 5: Some Moai
1. A Short Story of Easter Island

Why is it mysterious?

- The late stone age Polynesian culture found on Easter Island in 1722 was incapable of creating such monumental architecture.
- First, the culture seemed too poor to support a large artisan class devoted to carving statues.
- Second the statues were moved substantial distances from the island’s unique quarry to their destinations, but the population, estimated at about 3,000 in 1722, seemed too small to move the larger statues,
- No knowledge of tools such as levers, rollers, rope, and wooden sleds.
- The island in 1722 had no trees suitable for making such tools.
- Local residents had no knowledge of how to move the statues.
1. A Short Story of Easter Island

The Polynesian civilization in place at the time of first European discovery in 1722 was much poorer and much less populous than it had been a few hundred years earlier.

A economic story of rising wealth and rising population, followed by decline (collapse).

Easter Island suffered a sharp decline after perhaps a thousand years of apparent peace and prosperity.

The population rose well above its long-run sustainable level and subsequently fell in tandem with disintegration of the existing social order and a rise in violent conflict.

The islanders degraded their environment to the point that it could no longer support the population and culture it once had.
1. A Short Story of Easter Island

Archeological records

- Easter Island was first settled by a small group of Polynesians about or shortly after 400 A.D. (*Anno Domini = après J.C.*)
- The pollen record obtained from core samples and dated with carbon dating methods shows that the island supported a great palm forest at this time.
- In the years following initial settlement, one important activity was cutting down trees, making canoes, and catching fish.
- Archaeological record shows a high density of fish bones during this early period.
- Wood was also used to make tools and for firewood, and the forest was a nesting place for birds that the islanders also ate.
- The population grew rapidly and was wealthy in the sense that meeting subsistence requirements was relatively easy.
- This relative high level of wealth left resources to devote to other activities including, as time went on, carving and moving statues.
1. A Short Story of Easter Island
The collapse

- Noticeable forest reduction is evident in the pollen record by about 900 A.D.
- Most of the statues were carved between about 1100 and 1500.
- By about 1400 the palm forest was entirely gone.
- Diet changed for the worse as forest depletion became severe, containing less fish (and thus less protein) than earlier.
- Loss of forest cover also led to reduced water retention in the soil and to soil erosion (wind), causing lower agricultural yields.
- Population probably peaked at about 10,000 sometime around 1400 A.D., then began to decline.
- The period 1400 to 1500 was a period of falling food consumption and initially active, but subsequently declining, carving activity.
- Carving had apparently ceased by 1500.
1. A Short Story of Easter Island
The collapse (2)

- Around 1500, a new tool called “mata’a” enters the archaeological record.
- This tool resembles a spearhead and is likely to be a weapon.

Figure 6: Mata’a
1. A Short Story of Easter Island
The collapse (3)

- In addition, many islanders began inhabiting caves and fortified dwellings.
- There is also strong evidence of cannibalism at this time.
- Numbers of house sites in the coastal lowlands, where almost everybody lived, declined by 70% from peak values around 1400-1600 to the 1700s, suggesting a corresponding decline in numbers of people.
- With the exhaustion of resources, the island has entered into a period of social collapse, violence and population decline.
1. A Short Story of Easter Island

The collapse (4)

- Captain Cook in 1774 described the islanders as “small, lean, timid, and miserable.”
1. A Short Story of Easter Island
The collapse (5)

Figure 8: Demographic and ecological dynamics Bahn and Flenley [1992]
1. A Short Story of Easter Island
The Rapa Nui ecodisaster: A message for our future?

- The human population of Rapa Nui seems to have caused an environmental catastrophe leading to a demographic, economic and social collapse.
- An example of externalities caused by economic development: the person who cut the last tree might have known that it was the last one, but did not take into consideration that no further trees will grow.
- Why did environmental degradation lead to population overshooting and decline on Easter Island, but not on the other major islands of Polynesia?
- Let us study a dynamic economic model of renewable resource that can explain the facts (Brander and Taylor [1998])
2. A Model for the dynamics of Easter Island

Renewable resource dynamics

- The resource at time $t$ is $S(t)$
- Resource = the ecological complex consisting of the forest and soil
- $G(S(t)) = \text{natural growth}$
- $H(t) = \text{harvest}$
- Law of motion of the resource:

$$\dot{S}(t) = \frac{dS(t)}{dt} = G(S(t)) - H(t)$$
2. A Model for the dynamics of Easter Island

Renewable resource dynamics

- $G$ is assumed to be a logistic function:

$$G(S(t)) = rS(t) \left(1 - \frac{S(t)}{K}\right)$$

- $K$ = “carrying capacity” = maximum size of the resource
- $r$ = regeneration rate.
- Figure 9 is an example with $K = 10$ and $r = 20\%$ (black) or $r = 40\%$ (gray)
2. A Model for the dynamics of Easter Island

Renewable resource dynamics

Figure 9: $G(S)$ (panel (a)) and $S(t)$ starting from $S(0) = 5$ (panel (b))
2. A Model for the dynamics of Easter Island

Inputs and outputs

- The economy produces and consumes two goods.
- $H(t)$ is the harvest of the renewable resource,
- $M(t)$ is some aggregate "other good."
- In the case of Easter Island:
  - harvest is food
  - $M(t)$ is tools, housing, artistic output (including monumental architecture), household production, etc.
- good $M$ is the numéraire (price is 1)
- price of food is $p(t)$
2. A Model for the dynamics of Easter Island

Technologies and wage

- Production done with the resource and labor $L$
- Assumption: $L(t) = \text{population}$
- Technology for food (supply): $H^s(t) = \alpha S(t)L_H(t)$
- Technology for the other good: $M^s(t) = L_M(t)$
- Wages $w_H(t)$ and $w_M(t)$
- There is free disposal of the resource (all the production of food goes to the workers), so that

$$w_H(t) = p(t)\frac{H^s(t)}{L_h(t)} = p(t)\alpha S(t)$$

- Perfect competition on the labor market for the M good:

$$w_M(t) = \frac{dM^s(t)}{dL_M(t)} = 1$$
2. A Model for the dynamics of Easter Island

Preferences

- Individuals live one period and work 1 unit
- Fertility decisions are not derived from optimal behavior (as opposed to Lecture 1). Population dynamics will be described later.
- \[ u = h(t)^\beta m(t)^{1-\beta} \]
- By arbitrage, we will have \( w_H(t) = w_M(t) = w(t) \)
- Budget constraint: \( p(t)h(t) + m(t) = w(t) \times 1 \)
- Optimal solution (*details in class*)
  \[ h^d(t) = \beta \frac{w(t)}{p(t)} \]
  \[ m^d(t) = (1 - \beta)w(t) \]
- so that in the aggregate \( H^d(t) = \beta \frac{w(t)L(t)}{p(t)} \) and \( M^d(t) = (1 - \beta)w(t)L(t) \)
2. A Model for the dynamics of Easter Island

Price of the resource

- We have $w_H(t) = p(t)\alpha S(t)$.
- In equilibrium, $w_H(t) = w(t)$.
- Therefore $p(t) = \frac{w(t)}{\alpha S(t)}$. 
2. A Model for the dynamics of Easter Island

Equilibrium

- Supply of labor: \( L(t) \)
- Demand of labour: \( L_H(t) + M(t) \)
- Supply of food: \( H^s(t) = \alpha S(t) L_H(t) \)
- Demand of food: \( H^d(t) = \beta \frac{w(t)L(t)}{p(t)} \)
- Wage in the food sector: \( p(t) = \frac{w(t)}{\alpha S(t)} \)
- Wage in the “other sector”: \( w(t) = 1 \)
From the previous equations (details in class):

\[ H(t) = \alpha \beta S(t)L(t) \]

As \( L(t) \) is given at date \( t \), we obtain \( S(t) \)

The dynamics of the resource is given by

\[ \dot{S}(t) = \frac{dS(t)}{dt} = rS(t) \left( 1 - \frac{S(t)}{K} \right) - \alpha \beta S(t)L(t) \]

See Figure 10.
2. A Model for the dynamics of Easter Island

Equilibrium

Figure 10: Resource dynamics for a fixed population
2. A Model for the dynamics of Easter Island

Demographics

- mortality rate $d$ is exogeneous
- fertility rate is $b + F(t)$
- $F(t) = \frac{\phi H(t)}{L(t)}$ (fertility is increasing with the availability of resources.
- $b - d < 0$: without resources, population tends to zero.
- The growth rate of population is given by:

\[
\frac{dL(t)}{dt} = (b - d + F(t)) \times L(t)
\]

- See Figure 11
- Using $H(t) = \alpha \beta S(t)L(t)$, we can rewrite the equation of the population dynamics as

\[
\frac{dL(t)}{dt} = (b - d + \phi \alpha \beta S(t)) \times L(t)
\]
2. A Model for the dynamics of Easter Island

Equilibrium

Figure 11: Population dynamics for a fixed resource

\[
\frac{dL}{dt} = \frac{b-d+F}{L}\]

\[
b-d+F = b-d + \phi \alpha \beta S \quad \text{(net fertility)}
\]

Panel B: Population Dynamics

Resource Stock, S

b-d

S*

0

(dL/dt)/L
2. A Model for the dynamics of Easter Island

Equilibrium dynamics

The system dynamics is therefore given by the following pair of differential equations:

\[
\begin{align*}
\frac{dS(t)}{dt} &= rS(t) \left(1 - \frac{S(t)}{K}\right) - \alpha \beta S(t)L(t) \quad (A) \\
\frac{dL(t)}{dt} &= (b - d + \phi \alpha \beta S(t)) \times L(t) \quad (B)
\end{align*}
\]
2. A Model for the dynamics of Easter Island

Steady States

- A steady state satisfies $\dot{S}(t) = \dot{L}(t) = 0$
- Looking at the pair of differential equations we get the following proposition (see details in class)

Proposition 1

*There are three possible steady states:*

1. “unoccupied or virgin island”: $L = 0$ and $S = K$
2. “collapsed island”: $L = 0$ and $S = 0$
3. “sustainable island”:

\[
\begin{align*}
S &= \frac{d-b}{\phi \alpha \beta} \\
L &= \frac{r}{\alpha \beta} \left(1 - \frac{d-b}{\phi \alpha \beta K}\right)
\end{align*}
\]
2. A Model for the dynamics of Easter Island

Interior Steady State

\[
\begin{align*}
S &= \frac{d-b}{\phi\alpha\beta} \\
L &= \frac{r}{\alpha\beta} \left(1 - \frac{d-b}{\phi\alpha\beta K}\right)
\end{align*}
\]

- We can do some comparative statics for the “sustainable island” case, which is the interior stay state:

**Proposition 2**

*The interior steady state stock of resources $S$:*

1. *rises if the mortality rate rises, the birth rate falls, or fertility responsiveness falls*
2. *falls if there is technological progress in harvesting;*
3. *is unaffected by changes in the intrinsic resource regeneration rate, $r$, or carrying capacity, $K$.***
2. A Model for the dynamics of Easter Island

Interior Steady State (2)

\[
\begin{align*}
S &= \frac{d-b}{\phi \alpha \beta} \\
L &= \frac{r}{\alpha \beta} \left(1 - \frac{d-b}{\phi \alpha \beta K}\right)
\end{align*}
\]

Proposition 3

At the interior steady state, population \( L \):

1. rises equiproportionately with an increase in the intrinsic rate of resource growth, \( r \);
2. falls when harvesting technology improves if \( S < K/2 \) and rises if \( S > K/2 \);
3. falls when the taste for the resource good rises if \( S < K/2 \) and rises if \( S > K/2 \);
4. rises if the carrying capacity of the environment rises.
2. A Model for the dynamics of Easter Island

Interior Steady State (3)

\[
\begin{align*}
S &= \frac{d-b}{\phi \alpha \beta} \\
L &= \frac{r}{\alpha \beta} \left(1 - \frac{d-b}{\phi \alpha \beta K}\right)
\end{align*}
\]

- The interior steady state exists only if \( S < K \), as the level of resource cannot exceed \( K \)
- The condition on parameters for existence is therefore

\[ \frac{d-b}{\phi \alpha \beta} < K \]
2. A Model for the dynamics of Easter Island
Global dynamics

- We will not prove this result:

**Proposition 4**

*When an interior steady state exists, the global behavior of the system is as follows:*

1. *if* $L > 0$ *and* $S = 0$, *the system approaches the steady state with* $L = 0$ *and* $S = 0$;
2. *if* $L = 0$ *and* $S > 0$, *the system approaches the steady state with* $S = K$ *and* $L = 0$;
3. *if* $S > 0$ *and* $L > 0$, *then the system converges to the interior steady state.*
3. Applying the Model to Easter Island

Discretization

\[
\begin{cases}
\frac{dS(t)}{dt} = rS(t) \left(1 - \frac{S(t)}{K}\right) - \alpha\beta S(t)L(t) \quad (A) \\
\frac{dL(t)}{dt} = (b - d + \phi\alpha\beta S(t)) \times L(t) \quad (B)
\end{cases}
\]

- We first rewrite the differential system in discrete time.
- Note that it is also possible to simulate differential equations

\[
\begin{cases}
\frac{S(t+\Delta)-S(t)}{\Delta} = rS(t) \left(1 - \frac{S(t)}{K}\right) - \alpha\beta S(t)L(t) \quad (A) \\
\frac{L(t+\Delta)-L(t)}{\Delta} = (b - d + \phi\alpha\beta S(t)) \times L(t) \quad (B)
\end{cases}
\]
3. Applying the Model to Easter Island

Discretization (2)

\[
\begin{align*}
\frac{S(t+\Delta)-S(t)}{\Delta} &= rS(t) \left(1 - \frac{S(t)}{K}\right) - \alpha \beta S(t) L(t) \quad (A) \\
\frac{L(t+\Delta)-L(t)}{\Delta} &= (b - d + \phi \alpha \beta S(t)) \times L(t) \quad (B)
\end{align*}
\]

- Use the change of variable \( t_\tau = \Delta \tau \) for \( \tau = 1, 2, \ldots \)
- Write \( x(t_\tau) = x_\tau \)
- \( x_\tau \) is now a discrete time variable and we can rewrite the system as:

\[
\begin{align*}
S_{\tau+1} &= \left((1 + r \Delta) - \left(\frac{r \Delta}{K}\right) S_\tau - \alpha \beta \Delta L_\tau\right) S_\tau \quad (A) \\
L_{\tau+1} &= \left((1 + (b - d) \Delta) + \phi \alpha \beta \Delta S_\tau\right) L_\tau \quad (B)
\end{align*}
\]
3. Applying the Model to Easter Island

Parameter choice

- The unit of time is a decade (10 years)
- $K = 12000$ units (arbitrary)
- $K$ is chosen as the initial resource size: $S_0 = K$ (The forest has been in place for around 37000 years before the Polynesian colonization)
- Length of a period: $\Delta = 10$, meaning that one simulation period corresponds to $\Delta$ decades, i.e. 100 years.
- Labor productivity: $\alpha = 0.00001$ : if $S = K$, a household could provide its subsistence consumption (the amount just necessary to reproduce itself) in about 20 percent of its available labor time. Accordingly, there is considerable surplus on the island when the resource stock is large.
- Taste for food: $\beta = 0.4$ : also equal to the share of labor devoted to harvesting.
3. Applying the Model to Easter Island

Parameter choice (2)

- Intrinsc growth of the resource $r = 0.04$ : 4% per decade when the stock of resource is small (no congestion)
- Demographics : $b - d = -0.1$ : Absent of resource, population declines by 10% per decade.
- Demographics : $\phi = 4$ : Population grows if $S > K/2$ and decreases otherwise.
- Initial population: $L_0 = 40$
3. Applying the Model to Easter Island

Simulation 1

Figure 12: The dynamics of population and resource
3. Applying the Model to Easter Island

What is different in Easter Island?

- Easter Island is not different than other islands in terms of tastes, demographics or technology.
- One difference with other islands: the palm tree that grew on Easter Island happened to be a very slow-growing palm.
- Coconut trees, that one finds on other polynesian islands, grow faster.
- Assume that the renovation rate is $r = 0.3$ (instead of 0.04)
3. Applying the Model to Easter Island

Simulation 2: Fast growing trees

Figure 13: The dynamics of population and resource
3. Applying the Model to Easter Island

The 12 “mystery islands”

- There are 12 so-called “mystery islands” in Polynesia.
- These islands were once settled by Polynesians but were unoccupied at the time of European discovery.
- Assume that an island has a small carrying capacity $K$: there is not enough resources to maintain population.
- Population is driven to 0.
- Let’s simulate the model with $K = 4000$ instead of 12000.
- In such a case, an interior steady state does not exist. Population converges to zero and the island becomes again a virgin island.
3. Applying the Model to Easter Island

Simulation 3: “A Mystery Island”

Figure 14: The dynamics of population and resource
4. Summary

- Complex dynamics of population and resources
- Growth is not always sustainable in the long run with limited resources
- We have developed a model of the joint dynamics of population and resources
- The model can explain collapses, disparition of population or sustained positive population.
5. References

▶ Paul Bahn and John Flenley, 1992, “Easter Island, Earth Island”, Thames & Hudson Ltd.