2018-2019 – Econ 0107 – Macroeconomics I

Lecture 7 : Ricardian Equivalence

(Chapter 10 in Ljungqvist & Sargent)

Franck Portier
F.Portier@UCL.ac.uk

University College London

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- Key idea: the timing of lump taxes does not matter $\rightsquigarrow$ equivalence of the debt/lump taxes timing
- This is the equivalent in macro of the Modigliani-Miller theorem.
- Foramlly presented by Barro, JPE, 1974
- It means that the "keynesian multiplier $\Delta T = \Delta B/P$" (*cut in taxes financed by public debt*) is fallacious.
- I present the model, then state the result and discuss it.

- N identical households

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

  with all good properties, including $\lim_{c \downarrow 0} u'(c) = +\infty$
- No uncertainty
- The household can invest in a single risk-free asset bearing a fixed gross one-period rate of return $R > 1$: it is a loan to foreigners or to the government.
- 1 unit of b_{t+1} is a piece of paper that is sold R^{-1} units of good in period t and that promises 1 units of good in $t + 1$.
- $b > 0$ means that the Hh is net creditor, $b < 0$ net borrower.

- The time t budget constraint (BC) is

$$c_t + R^{-1} b_{t+1} \leq y_t + b_t \tag{2}$$

  with b_0 given.
- Assume that $R\beta = 1$ and that $\{y_t\}_{t=0}^{\infty}$ is a given nonstochastic nonnegative endowment sequence with $\sum_{t=0}^{\infty} R^{-t} y_t < \infty$.
- The extent to which Ricardian Equivalence holds depends on households' access to financial markets. We explore two possibilities.
- The first one is that the household can lend but not borrow: $b_t \geq 0$ for all t.
- The second one is that the household cannot borrow more that it is feasible to repay: $b_t \geq \widetilde{b}_t$ for all t.
- I will (loosely) refer to this case as the no financial constraint case.

- This maximum amount $\widetilde{b}_t$ is computed by setting $c_t = 0$ for all t in (2) and solving forward:

$$\widetilde{b}_t = -\sum_{j=0}^{\infty} R^{-j} y_{t+j} \tag{3}$$

where the following transversality condition have been imposed:

$$\lim_{T \to \infty} R^{-T} b_T = 0 \tag{4}$$

- This $\widetilde{b}_t$ is referred to as the *natural debt limit* and the alternative restriction is

$$b_t \geq \widetilde{b}_t \tag{5}$$

- The the typical intertemporal household problem is here to maximize (1) s.t. (2) and $b_{t+1} \geq 0$.

$$\max_{b_{t+1}, c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) \right]$$

- The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t = \beta \lambda_{t+1} \\ \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) = 0 \\ \lambda_t \geq 0 \end{cases}$$

- which implies:

$$u'(c_t) = \beta R u'(c_{t+1}) \quad \forall t \geq 0$$

- The the typical intertemporal household problem is here to maximize (1) s.t. (2) and $b_{t+1} \geq 0$.

$$\max_{b_{t+1}, c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) + \mu_t b_{t+1} \right]$$

- The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t + \mu_t = \beta \lambda_{t+1} \\ \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) = 0 \\ \mu_t b_{t+1} = 0 \\ \lambda_t \geq 0 \\ \mu_t \geq 0 \end{cases}$$

- which gives

$$u'(c_t) \geq \beta R u'(c_{t+1}) \quad \forall t \geq 0 \tag{6a}$$

$$u'(c_t) > \beta R u'(c_{t+1}) \text{ implies } \quad b_{t+1} = 0 \tag{6b}$$

- with $\beta R = 1$, this becomes $c_t = c_{t+1}$ when the consumer is not constrained ($b_{t+1} \geq 0$) and $c_{t+1} < c_t = y_t + b_t$ when she is constrained ($b_{t+1} = 0$).

Example 1 : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_h, y_l, y_h, y_l, ...\}$ with $y_h > y_l > 0$, $b_t \geq 0 \ \forall t$.

Example 2 : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, y_h, ...\}$ with $y_h > y_l > 0$

Example 3 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$

Example 4 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$ and $b_t \geq \widetilde{b}_t$ is imposed.

Example 5 : $b_0 = 0$, $y_t = \lambda^t$ where $1 > \lambda$ and $b_t \geq \widetilde{b}_t$ is imposed.

- The Gvt purchases a stream $\{g_t\}_{t=0}^{\infty}$ per household, imposes a stream of lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$ and is subject to the BC:

$$B_t + g_t = \tau_t + R^{-1} B_{t+1} \tag{8}$$

- B_t is a one-period debt due at t and denominated in period t consumption good. The Gvt is allowed to borrow.
- Ruling out Ponzi schemes ($\lim_{T\to\infty} R^{-T} B_{T+1} = 0$), one gets from solving (8) forward:

$$B_t = \sum_{j=0}^{\infty} R^{-j} \left(\tau_{t+j} - g_{t+j}\right) \tag{9}$$

- The household's BC (2) becomes

$$c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t \tag{10}$$

Solving forward and using the transversality condition:

$$b_t = \sum_{j=0}^{\infty} R^{-j} (c_{t+j} + \tau_{t+j} - y_{t+j}) \tag{11}$$

and the natural debt limit is

$$\widetilde{b}_t = \sum_{j=0}^{\infty} R^{-j} (\tau_{t+j} - y_{t+j}) \tag{12}$$

- Note that the debt limit is greater (I mean more binding) with positive taxes.

Definition 1

Given initial condition (b_0, B_0), an **equilibrium** is a household plan $\{c_t, b_{t+1}\}$ and a government policy $\{g_t, \tau_t, B_{t+1}\}$ such that (a) the government plan satisfies the government BC (8) and (b) given $\{\tau_t\}$, the household plan is optimal.

Proposition 1

Suppose that the natural debt limit prevail. Given initial conditions (b_0, B_0)*, let* $\{\overline{c}, \overline{b}_{t+1}\}$ *and* $\{\overline{g}_t, \overline{\tau}_t, \overline{B}_{t+1}\}$ *be an equilibrium. Consider any other tax policy* $\{\widehat{\tau}_t\}$ *satisfying*

$$\sum_{t=0}^{\infty} R^{-t} \widehat{\tau}_t = \sum_{t=0}^{\infty} R^{-t} \overline{\tau}_t \tag{13}$$

Then $\{\overline{c}_t, \widehat{b}_{t+1}\}$ *and* $\{\overline{g}_t, \widehat{\tau}_t, \widehat{B}_{t+1}\}$ *is also an equilibrium where*

$$\widehat{b}_t = \sum_{j=0}^{\infty} R^{-j} \left(\overline{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}\right) \qquad \text{and} \qquad \widehat{B}_t = \sum_{j=0}^{\infty} R^{-t} \left(\widehat{\tau}_{t+j} - \overline{g}_{t+j}\right)$$

In words, the timing of taxes and debt does not matter. What matters is their present value.

- We need to show *(i)* that the consumption plan $\{\overline{c}_t\}$ and the adjusted borrowing plan $\{\widehat{b}_{t=1}\}$ solve the household's optimum problem and *(ii)* that the altered government tax and borrowing plans continue to satisfy the government's BC.
- $\rightsquigarrow$ *(i)* At time 0, the household face a single intertemporal budget constraint (this is true under the natural debt limit)

$$b_0 = \sum_{t=0}^{\infty} R^{-t}(c_t - y_t) + \sum_{t=0}^{\infty} R^{-t}\tau_t$$

- Therefore, the household's optimal consumption plan does not depend on the timing of taxes, but only on their net present value $\rightsquigarrow \{\overline{c}_t\}$ is still feasible and optimal.
- Having $\{\overline{c}_t\}$, we can construct the sequence of $\{\widehat{b}_{t+1}\}$ by solving the household 's BC (10) forward to obtain (14). To do so, we use a transversality condition $\lim_{T\to\infty} R^{-T}\widehat{b}_{T+1} = 0$. Let's check that it is satisfied if the transversality condition is satisfied for the original borrowing plan:

Proof of Proposition 1

- In an period $k-1$, solving the BC (10) backwards yields

$$b_k = \sum_{j=1}^{k} R^j \left(y_{k-j} - \tau_{k-j} - c_{k-j}\right) + R^k b_0$$

- which gives

$$\overline{b}_k - \widehat{b}_k = \sum_{j=1}^{k} R^j \left(\widehat{\tau}_{k-j} - \overline{\tau}_{k-j}\right)$$

- which is also, by $\times R^{1-k}$

$$R^{1-k}\left(\overline{b}_k - \widehat{b}_k\right) = R\sum_{t=0}^{k-1} R^{-t}\left(\widehat{\tau}_t - \overline{\tau}_t\right)$$

- The limit of the RHS is zero when $k \to \infty$ because of (13). Then, given that $\{\overline{b}_{t+1}\}$ satisfies the TC, $\{\widehat{b}_{t+1}\}$ does.

$\hookrightarrow$ *(ii)* Let us show now that the altered government tax and borrowing plans satisfy the government BC. This BC is given by

$$B_0 = \sum_{t=0}^{\infty} R^{-j} \tau_t - \sum_{t=0}^{\infty} R^{-j} g_t$$

From (13), we now that the BC is still satisfied. The sequence of $\widehat{B}_{t+1}$ can then be recovered by solving forward this BC at every period t. $\square$

- The former proposition relies on the fact that the household can undo what the government does by using financial markets.
- This neutrality results does not hold any more in the no-borrowing constraint case.
- Now, a change in the timing of taxes can cause a previously non binding constraint binding.
- We have only a weak form of neutrality

Proposition 2

Consider an initial equilibrium with consumption path $\{\overline{c}\}$ *in which* $b_{t+1} > 0$ *for all* $t \geq 0$. *Let* $\{\overline{\tau}_t\}$ *be the tax rate in the initial equilibrium, and let* $\{\widehat{\tau}_t\}$ *be any other tax rate sequence with same present value and for which*

$$\widehat{b}_t = \sum_{j=0}^{\infty} (\overline{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}) \geq 0 \qquad (\star)$$

for all $t \geq 0$. *Then* $\{\overline{c}_t\}$ *is also an equilibrium allocation for the* $\{\widehat{\tau}_t\}$ *sequence.*

- If $(\star)$ is satisfied, then the household can undo the change in the government tax and borrowing plan without hitting the no-borrowing constraint.
- The sequence $\{\overline{c}_t\}$ is therefore feasible, and then, one can proceed as in the preceding proof. $\square$

4. Linked Generations Interpretation

- Often the Ricardian equivalence results is dismissed as irrealistic because the time horizon of some households is shorter than the government one ("I'll be dead before they start raising taxes to pay back public debt $\rightsquigarrow$ for me, government bonds are net wealth")
- Barro was the first to show that this is not true if generations are linked by bequests.
- The model with borrowing constraints can be reinterpreted in such a way:

4. Linked Generations Interpretation

- Assume that there is a sequence of one-period-lived agents, that value consumption and the utility of its unique offspring:

$$u(c_t) + \beta V(b_{t+1})$$

where b_{t+1} is the amount of bequest that is left to generation $t+1$ and V is the maximized utility of a time $t+1$ agent, recursively defined as

$$V(b_t) = \max_{c_t, b_{t+1}} \{u(c_t) + \beta V(b_{t+1})\} \quad (16)$$

$$\text{s.t.} \quad c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t \quad (17)$$

with $b_{t+1} \geq 0$

- This model consumption equilibrium allocations are identical to those of the infinitely-lived one with a no-borrowing constraint. Therefore, the weak version of the Ricardian Equivalence theorem holds.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Intergenerational Redistribution

- As I said before, if tax cut or the current generation are financed by tax increase on the next generation, Ricardian Equivalence does not hold
- This is not true if there is "perfect" intergenerational altruism.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Capital Market Imperfections

- Again, I have shown before that only a weak form of Ricardian Equivalence holds if there is a no-borrowing constraint.
- It is also the case if there is a wedge between creditor's interest rate and debtor's one.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Distortionary Taxes

- If taxes are distortionary, then their timing affect household's decisions.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Income Uncertainty

- Government debt might affect consumer's perception of the risks they face, and therefore affects their current consumption.
- Assume that taxes are levied as a function of income, and that future income is uncertain.
- Assume that the government cuts taxes today, issues debt today and raises income taxes in the future to pay off the debt.
- In such a case, consumer's expected lifetime income is unchanged, but the uncertainty they face is reduced. If they have precautionary savings, this reduction in uncertainty will reduce those savings and therefore foster consumption.

6. Is the Ricardian Equivalence (or the lack of) Empirically Relevant?

- Difficult to test directly. The Ricardian argument does not render all fiscal policy irrelevant.
- For example, if the government cut taxes today and households expect this tax cut to be met with future cuts in useless government expenditures instead of future tax increases, households' permanent income increases and so does consumption. $\rightsquigarrow$ but one does not observe directly expectations...
- Some assumptions or implications of the Ricardian Equivalence result can be tested.

6. Is the Ricardian Equivalence (or the lack of) Empirically Relevant?

Testing Assumptions

- It has been shown that consumers do not smooth consumption as much as Permanent Income theory predicts $\rightsquigarrow$ there are liquidity constraints, financial imperfections,...

6. Is the Ricardian Equivalence (or the lack of) Empirically Relevant?

Testing Implications for Consumption

- In a consumption equation

$$C = f(\text{income, wealth, fiscal policy, taxes, public debt,...})$$

  the coefficients on taxes and public debt should be zero.
- but a lot of implementation problems (expectations (suppose that the current level of taxes affect expectations about future government expenditures) , simultaneity (shocks to consumption might affect fiscal policy),...)
- Using a Euler equation approach, one might overcome some of those difficulties (expectations), but results are not conclusive.

6. Is the Ricardian Equivalence (or the lack of) Empirically Relevant?

Testing Implications for Interest Rates

- A debt-financed reduction in government revenues should not affect interest rates, while it should increase it according to the traditional view (say IS-LM).
- Again, the problem is that it is hard to get rid of the changes in expectations $\rightsquigarrow$ difficult to conclude.

7. Fiscal Adjustment in the OECD

- Since the 90's, a literature has systematically studied some episodes of fiscal adjustment (meaning a reduction of budget deficit)
- According to the traditional view, such episodes of fiscal restraints should create recessions (IS-LM, AD-AS)
- Some episodes (Ireland, Sweden, Denmark) show it is not the case
- Some more general lessons can be drawn from an extensive study of fiscal adjustment episodes in the Oecd
- See the work of Alesina, Ardagna, Giavazzi, Perotti and Pagano

7. Fiscal Adjustment in the OECD

Denmark (1983-86) : "expansionary stabilization"

- From 1980 to 1982, large increase of public debt, from 29% to 65% of GDP + high interest rates (22% for long run rates in oct 1982) + high fiscal deficit
- in 1982, a conservative coalition is elected: within 4 years, the primary deficit increases by 15 point, and the debt/gdp ration decreases;
- The danish krone is pegged with the DM;
- Growth is large: 3.6% per year from 1983 to 1986.

Table 1: Fiscal adjustment in Denmark and Ireland (average annual growth rates)

	Denmark		Ireland		
	1979-82	1983-86	1979-81	1982-84	1987-89
Gouvernment					
Public Consommation	4	0.9	4	0.7	-3.7
Public Investment	-9.4	-1.1	6.5	-6	-3.7
Private Sector					
Disposable Income	2.6	-0.3	1	-1.2	3.1
Consumption	-0.8	3.7	2.2	-1.2	3.1
Investment	-2.9	12.7	7.2	-4.7	6.7
Gdp	1.3	3.6	2.7	0	3.7

- In 1981, Primary deficit ($G - T$, i.e. *before* interest payments) is 8.4% of GDP, and interest payments on public debt represent 8.3%;
- From 1982 to 1984, the primary deficit is reduced by more than 7 points (most of it by increase in taxes) , and it is announced that the Irish Pound will peg the DM (to commit to disinflation)
- One observes a large contraction of private demand: consumption drops by 7% en 1982, and stays flat the 2 next years, and investment also drops.

- In 1987, a second plan pf fiscal adjustment is implemented.
- The reduction of fiscal deficit is mainly obtained by reducing govt consumption (that includes civil servant wages) and public investment.
- It is accompanied with a boom: primary deficit is reduced by 7 points in 2 years, growth increases and the debt/Gdp ratio goes down for the first time since 1970
- This adjustment is accompanied with a large devaluation that pushed exports up.

Sweden (1990-94): A Non-Keynesian fiscal expansion :

- 1990-93 is the worst recession of the post-WWII period in Sweden.
- 500 000 jobs are destroyed and unemployment reaches 14%
- Primary deficit increases by 16.2 GDP points (Half of it being the consequence of the recession, the other half being the discretionary decision of the govt)
- Nevertheless, one observes a dramatic recession (larger than without this fiscal adjustment plan ?)

Sweden (1990-94): A Non-Keynesian fiscal expansion :

Table 2: Sweden (1990-94) (% of GDP and Annual Growth rates)

	1990	1991	1992	1993	1994
Public Revenues	63.3	60.3	59.8	59.2	58.4
Public Expenditures	59	61.4	67.2	71.1	66
Primary Deficit	4.3	-1.1	-7.4	-11.9	-7.6
Private Consumption Growth	-0.7	0	0.4	-0.7	0.1
Private Investment Growth	1.6	-10.8	-12.4	-21	-1.8

- "The design of fiscal adjustments", Alesina & Ardagna [2012]
- 21 OECD countries from 1970 to 2010
- Definition: A fiscal adjustment is either:
  1. a two year period in which the cyclically adjusted primary balance/GDP improves in each year and the cumulative improvement is at least two points of the balance/GDP ratio;
  2. a three or more year period in which the cyclically adjusted primary balance over GDP improves in each year and the cumulative improvement is at least three points of the balance/GDP ratio.
- 52 episodes of fiscal adjustments

- A period of fiscal adjustment is successful if the debt to GDP ratio two years after the end of a fiscal adjustment is lower than the debt to GDP ratio in the last year of the adjustment.
- 25 episodes of successful fiscal adjustments and 24 unsuccessful.
- The successful episodes are those based mostly upon spending cuts rather than tax increases.

- A period of fiscal adjustment is expansionary if real GDP growth during the adjustment period is higher than the average growth the country experienced in the two years before.
- 35 episodes are expansionary fiscal adjustments and 17 are contractionary.
- The expansionary episodes are those based mostly upon spending cuts rather than tax increases.