PROBLEM SET 8: FISCAL POLICIES IN A GROWTH MODEL

Note: Those problems are about the "growth model" aspect of the lecture (i.e. "RBC" models), not about fiscal policy.

PROBLEM I – A SIMPLIFIED REAL-BUSINESS-CYCLE MODEL WITH ADDITIVE TECHNOLOGY

Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual maximizes the expected value of $\sum_{t=0}^{\infty} u(C_t)/(1+\rho)^t$, $\rho > 0$. The instantaneous utility function, $u(C_t)$, is $u(C_t) = C_t - \theta C_t^2$, $\theta > 0$. Assume that C is always in the range where $u'(C)$ is positive.

Output is linear in capital, plus an additive disturbance: $Y_t = AK_t + e_t$. There is no depreciation; thus $K_{t+1} = K_t + Y_t - C_t$, and the interest rate is A. Assume $A = \rho$. Finally, the "disturbance" e time path is given by : $e_0 = 1$, $e_t = \phi e_{t-1}$, where $-1 < \phi < 1$. Its is assumed that K_0 is at the steady state level corresponding to $e = 0$.

1 – Find the first-order condition (Euler equation) relating C_t and C_{t+1}.

2 – Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess, what is K_{t+1} as a function of K_t and e_t?

3 – What is the dynamics of the C and K from 0 to $+\infty$?

4 – What are the effects of a one-time shock to ε on the paths of Y, K, and C?

PROBLEM II – AN ANALYTIC MODEL WITH LOG-LINEAR DEPRECIATION

Consider a model economy populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = Z_t K_t^\gamma N_t^{1-\gamma} \tag{1}$$

where K_t is capital, N_t labor input, and Z_t a deterministic technological parameter. All profits of the firm are distributed to the household. Capital evolves according to the log linear relation

$$K_{t+1} = A K_t^{1-\delta} I_t^\delta \qquad 0 < \delta \leq 1 \tag{2}$$

where δ accounts fro capital depreciation depreciation and where I_t is investment in period t.

The representative household works N_t, consumes C_t and invests I_t in period t. Preferences are given by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log C_t - V(N_t) \right] \tag{3}$$

where V is a convex function. Capital is accumulated by the household and rented to the firm.

Let κ denote the real rental rate of capital, P the price of the final good and W the nominal wage.

Finally, the time path of Z and K_0 are given.

We first assume that $\delta = 1$ (full depreciation).

1 – Write down the budget constraint of the household and the profit function of the firm

2 – Derive FOCs of the utility and profit maximization

3 – Define a competitive equilibrium of this economy

4 – Solve the model and show that N_t is constant along an equilibrium path.

5 – Derive a first order difference equation equation for log of output (given a time path for Z).

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We assume now that $\delta \in]0, 1[$.

6 – What is the economic meaning of equation (2)?

7 – We still denote κ_t the real return in t on investment I_{t-1}:

$$\kappa_t = \frac{\partial Y_t}{\partial I_{t-1}} \tag{4}$$

and note that the real price, in terms of current output, of capital to be transmitted to the next period is not one as when $\delta = 1$. We denote it as q_t (like Tobin's q), and it is equal to:

$$q_t = \frac{\partial K_{t+1}/\partial K_t}{\partial K_{t+1}/\partial I_t} \tag{5}$$

8 – Comment the new budget constraint of the household

$$C_t + I_t = \frac{W_t}{P_t} N_t + \kappa_t(I_{t-1} + q_{t-1} K_{t-1}) \tag{6}$$

9 – Solve for the competitive equilibrium of this economy and compute $\frac{I_t}{C_t}$.

10 – Compute the constant level of N.

11 – Derive the new difference equation followed by the log of output.

PROBLEM III – TECHNOLOGICAL CHANGE, PREFERENCE CHANGE AND THE ENDOGENEITY OF TFP

We consider here a variation of the simple analytical RBC model. We study a model economy $\mathcal{A}$ populated with a representative household and a representative firm. The firm has a Cobb-Douglas technology:

$$Y_t = e^{z_t} K_t^\gamma N_t^{1-\gamma} \tag{7}$$

where K_t is capital, N_t labor input, and e^{z_t} the exogenous Total Factor Productivity (TFP). All profits of the firm are distributed to the household. Capital evolves according to

$$K_{t+1} = I_t \tag{8}$$

where I_t is investment in period t.

The representative household works N_t and consumes C_t. Preferences are given by

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log C_t - e^{\chi_t} N_t \right] \tag{9}$$

where χ_t is a preference shifter. Capital is accumulated by the household and rented to the firm.
Let κ_t denote the real rental rate of capital, P_t the price of the final good and W_t the nominal wage. It is assumed that γ and β are between 0 and 1.
We finally assume that in this deterministic economy, K_0 is at the steady state level corresponding to $z = \chi = 0$.

1 – Write down the budget constraint of the household and the profit function of the firm

2 – Derive FOCs of the utility and profit maximization

3 – Define a competitive equilibrium of this economy

4 – Solve the model and show that the equilibrium process of output is $y_t = z_t + \gamma y_{t-1} - (1-\gamma)\chi_t$ (1) (dropping constants and with the notation $x = \log X$)

5 – We assume now $z_t = 0 \ \forall t \geq 0$ and that the time path of χ is given by : $\chi_0 = 1$, $\chi_t = 0 \ \forall t > 0$. What is the time path of $\log y_t$?

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We now consider an economy $\mathcal{B}$, in which the TFP is not exogenous at the aggregate level, but given by $e^{z_t} = \overline{Y}_t^{\theta} e^{x_t}$, where x is the exogenous part of TFP and $\overline{Y}_t^{\theta}$ acts as an externality. More precisely, $\overline{Y}_t$ is taken as given by firms and households, but one has at the competitive equilibrium $\overline{Y}_t = Y_t$. I is assumed that θ is between 0 and 1.

6 – What is the economic interpretation of this externality?

7 – Solve for the competitive equilibrium and give the equilibrium process of output (again in logs). Comment

8 – Assume that an economist observes the economy $\mathcal{B}$, with externality, but thinks that he is observing economy $\mathcal{A}$, and is therefore using equation (1) to measure TFP. Draw the economy dynamics when $\chi_t = 0 \; \forall t \geq 0$ and $\chi_0 = 1$, $\chi_t = 0 \; \forall t > 0$.

9 – How to interpret the positive correlation between observed TFP and output? Could such an economist believe (wrongly) that technological changes are driving part of the response of the economy? Discuss.

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PROBLEM IV – SIMULATING A DETERMINISTIC MODEL WITH DYNARE

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In this problem, the objective is to use DYNARE to reproduce the following figure of LJUNDQVIST & SARGENT's, Recursive Macroeconomic Theroy (4th edition), Chapter 11),
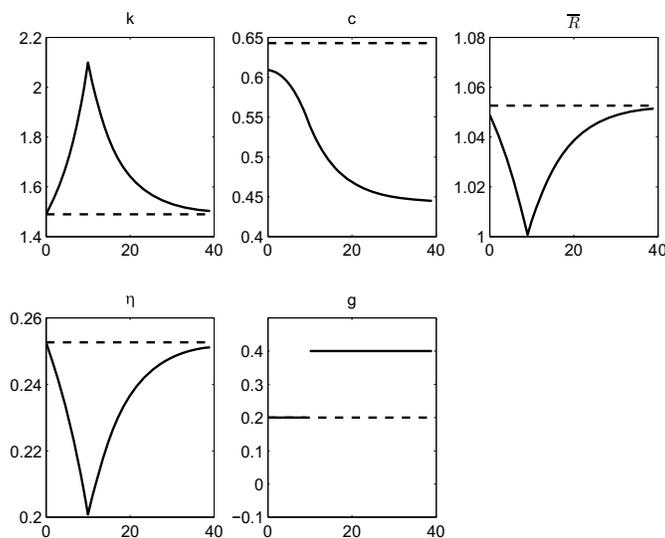

Figure 11.9.1: Response to foreseen once-and-for-all increase in g at $t = 10$. From left to right, top to bottom: $k, c, \bar{R}, \eta, g$. The dashed line is the original steady state.

as well as the two next figures in the textbook; and also to go a bit further.

To do so, it is useful to read Section 4 of BARILLAS, BHANDARI, COLACITO, KITAO, MATTHES, SARGENT & SHIN's "Practicing Dynare" [2010] (that you will find on the course page or at http://www.tomsargent.com/research/AP_tom16.pdf). That section is about replicating these figures. You will also find on the page of the course the DYNARE code for those three figures, as given by BARILLAS, BHANDARI, COLACITO, KITAO, MATTHES, SARGENT & SHIN and updated by me to be compatible with the current version of DYNARE (version 4.5.4). The original versions of these codes is there: http://www.tomsargent.com/research/examples.zip

In order to download, instal, etc ...DYNARE for MATLAB, see there: http://www.dynare.org. Good introductions to the use of DYNARE are the ones of ERIC SIMS (https://www3.nd.edu/~esims1/using_dynare_sp15.pdf) and WOUTER DEN HAAN (http://econ.lse.ac.uk/staff/wdenhaan/numerical/slidesdynare.pdf).

1 – Read and understand the codes `Figv3_1161_FP.mod`, `Figv3_1162_FP.mod` and `Figv3_1163_FP.mod`.

2 – Run those three codes to obtain figures 11.9.1 to 11.9.3 of LJUNDQVIST & SARGENT.

3 – Produce new figures when the shock is announced for period 20.

4 – Produce new figures when the increase in g follows a diffusion of the type displayed on the figure below.

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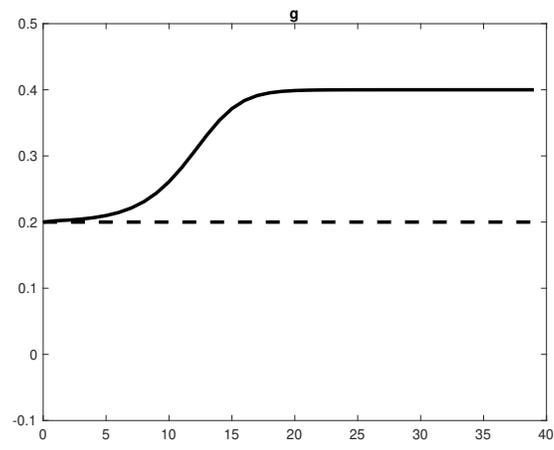
5 – Produce new figures when preferences are

$$u(c) = \frac{(c - \bar{c})^{1-\gamma}}{1 - \gamma},$$

with $\bar{c} = .4325$.