University College London, 2018-2019
Econ 3029 – Advanced Macro – Franck Portier

Problem set 1: (very) Long Run growth

Version 1.0 - 02/01/2019

Problem I – Discussing Maddison's estimates

The text in the appendix is a quite critical review of Maddison's historical estimates of income per capita. It is nevertheless a good starting point to get an idea of the methods and difficulties in constructing historical economic data.

1 – What does represent the level of 400 dollars at 1990 international prices in the work of Maddison?

2 – What is the main critique of Clark?

3 – According to Clark, to what $400 corresponds, and why is it not realistic?

4 – Explain the use of data on the evolution of humans height made by Clark.

5 – Why is urbanization an interesting statistics?

Problem II – The three stages of economic growth

In this problem, we will work with the dataset that was put together by Angus Maddison. On this page http://www.ggdc.net/maddison/oriindex.htm, download the Excel spreadsheet Statistics on World Population, GDP and Per Capita GDP, 1-2008 AD

1 – Take income per capita for the following set of countries: France, Portugal, Mexico, China, Morocco. For each country, plot the evolution of income per capita.

2 – Compute for each county average and total growth rate of per capita income for the following periods: 1-1700, 1700-1820, 1820-1913, 1913-2008. Discuss the results.

3 – Compute for each year the coefficient of variation of per capita income. Plot the evolution with time. Discuss. Note that the coefficient of variation is defined as Standard Deviation/Mean.

4 – Repeat questions 1 and 2 for another country of your choice. Discuss.

Problem III – The Malthusian Regime

The model is in discrete time. Consider the following model of joint determination of population and income per capita. The birth rate is given by:

$$b_t = \alpha_b + \beta_b y_t, \tag{1}$$

the mortality rate by:

$$m_t = \alpha_m - \beta_m y_t, \tag{2}$$

and the production function is:

$$Y_t = \alpha_y + \beta_y P_t. \tag{3}$$

Y stands for total production (or income), P for population and y for income per capita. It is assumed that all the population works and that there is non immigration nor emigration. All coefficients of the model are positive. It is assumed that $\alpha_m > \alpha_b$ and $\alpha_m - \alpha_b - \beta_y(\beta_b + \beta_m) > 0$.

1 – Discuss equations (1) to (3). Compute marginal and average productivity of labor. Comment.

2 – Compute the steady state level of total income, per capita income and population. Show graphically how those steady state values are determined.

3 – For the rest of the exercice, we assume $\alpha_b = \beta_y = 0$, $\beta_b = \beta_m = 0.5$, $\alpha_y = 1$ and $\alpha_m = \alpha$. What are the steady state levels for this configuration of parameters?

4 – Show that the model dynamics can be summarized by a first order difference equation in P_t (of the type $P_{t+1} = f(P_t)$).

5 – Study the convergence of population to its the steady state starting from a P_0 close to 0 for the following values of α: *(i)* $0 < \alpha < 1$, *(ii)* $\alpha = 1$, *(iii)* $1 < \alpha < 2$, *(iv)* $\alpha = 2$, *(v)* $\alpha > 2$.

Problem IV – Modeling mortality

We assume that mortality i.e. modeled as a rare event in continuous time. To do so, we make use of the exponential distribution. More precisely, we assume that life length is a continuous stochastic variable, whose probability density function is given by

$$f(L) = me^{-mL}$$

where $L \geq 0$ is life length and m is a positive parameter that we refers to as the mortality rate. The unit of time is the year and $m = 0.01$.

1 – Compute the cumulative distribution function of L, defined as $F(L) = P(L \leq \ell)$.

2 – Compute $P(L > \ell)$.

3 – Compute the probability that life lasts at least 10 years.

4 – Compute the probability that life lasts at least 80 years.

5 – Compute the probability for a person of age 10 that she lives at least until the age of 20. [Hint: use the Bayes formula $P(A|B) = P(A \cap B)/P(B)$]. Compare the result to the answer top question 3. Why is this model often called the "perpetual youth" model?

6 – Prove more generally the property of memorylessness of the distribution, *i.e.* $P(L > a + b|L > a) = P(L > b)$.

7 – Compute expected life length at birth and at age 50. Discuss.

Problem V – A model of unified growth in which adults consume food

We have studied in section 6 of Lecture 1 a model of unified growth. One unrealistic model assumption was that adults do not consume food, but only buy food for there children. In this problem, we remove this assumption and compute again the model equilibrium.

Adults like manufacturing goods m_t, adult food f_t and children n_t. Their utility is given by

$$u_t = m_t + \gamma \log n_t + \delta \log f_t.$$

As in the lecture, the manufacturing good is the numéra ire and one children costs one unit of food, so that the budget constraint of an adult in t is :

$$p_t n_t + p_t f_t + m_t = w_t.$$

1 – Solve the utility maximization problem of an adult and derive the demand for children and the demand for agricultural goods (food for chidden and for adults).

2 – Show that total demand for food is $\frac{(\gamma+\delta)}{p_t} L_t$.

As in the lecture, technology is for food $Y_t^A = A_t^\epsilon (L_t^A)^\alpha X^{1-\alpha}$, where X is land, assumed to be in fixed supply, and normalized to $X = 1$; L^A is agricultural labor and A is productivity on agriculture. ϵ and α are between 0 and 1. The technology for the manufacturing good is $Y_t^M = M_t^\epsilon L_t^M$. Note that we assume for simplicity that land is free disposal (land rent is zero). We finally assume Learning-By-Doing, so that $A_{t+1} - A_t = Y_t^A$ and $M_{t+1} - M_t = Y_t^M$.

3 – Write the equilibrium condition on the food market and derive from that condition an expression for the share of agricultural labor in total labor $\theta_t = \frac{L_t^A}{L_t}$. How is this expression different from the one in the lecture? Discuss.

4 – Give an expression for the wages in the two sectors.

5 – Why free movements of workers between sectors will equalize the wages? From the wage equality condition, derive an expression for p_t as a function of M_t, A_t, θ_t and L_t.

6 – Using the demand for children, derive n_t as a function of M_t, A_t, and L_t. How is this expression different from the one in the lecture? Discuss.

7 – Show that the long run level of population growth is the same than in the model of Lecture 5.

Problem VI – Simulation of the model of unified growth

Take the model of the last part of section 7 of Lecture 1 with $Y_t^A = \mu A_t^\epsilon (L_t^A)^\alpha X^{1-\alpha}$ and $Y_t^M = \delta M_t^\phi L_t^M$. The model solution is $n_t = \mu \left(\frac{\gamma}{\delta}\right)^\alpha \frac{A_t^\epsilon}{M_t^{\alpha\phi}(L_t)^{1-\alpha}}$ and $\theta_t = \left(\frac{n_t L_t^{1-\alpha}}{\mu A_t^\epsilon}\right)^{\frac{1}{\alpha}}$.

Assume that parameters are $\alpha = 0.8$, $\epsilon = 0.45$, $\phi = 0.3$, $\mu = 0.5$, $\delta = 1.5$, $\gamma = 3.4$ and initial values are $A_0 = 0.8$, $M_0 = 18.15$ and $L_0 = 0.014$. It is assumed that the length of the period is 25 years (one generation), so that growth factors needs to be taken at the power $1/25$ to be expressed in annual terms.

Use a spreadsheet (Excel, Numbers,...) or the scientific language of your choice (Matlab, Python, Julia, C++, Fortran,...) to reproduce the figures for time evolution of g_L, g_A, g_M and w.

Appendix : Review of Angus Maddison book *"Contours of the World Economy, 1-2030 AD: Essays in Macroeconomic History"* by Gregory Clark in Journal of Economic History, 2009

Book Reviews

GENERAL AND MISCELLANEOUS

Contours of the World Economy, 1–2030 AD: Essays in Macro-Economic History. By Angus Maddison. Oxford: Oxford University Press, 2007. Pp. xii, 418. $99.00, cloth; $42.95, paper.

Angus Maddison has labored fifty years to provide ever more comprehensive and distant estimates of economic growth. His earliest estimates of GDP began in 1870, the era of modern statistical offices and censuses, and mainly concerned the better documented economies of Europe and North America.[1] But as his fame has grown, Maddison has become ever bolder in the mysteries of his craft. His statistical empire now spans every corner of the globe, and extends over millennia. *Contours of the World Economy* projects a revised set of these estimates, for every country on earth, back to 1 AD.[2]

Maddison's numbers have found wide currency with economists analyzing long-run growth. Google Scholar records over 12,000 references to his works. Even the most speculative estimates have been employed in high-profile publications, as is discussed below. *Contours of the World Economy* is a magisterial summary of this life project, and a discussion of elements of the derivation of the earlier data. It gives detail down to the level of estimates of population and GDP per capita in Belgium in 1 AD. It has, for example, graphs such as one comparing U.K. and Indonesian GDP per capita, by year, 1500–2003 (p. 132). It reports that the poorest people who ever lived were the Nepalese, whose GDP per capita in 1820 and 1870 is estimated at only $397 (in 1990 $) (p. 176). It is also, as the subtitle "essays" warns, a somewhat disjointed explanation of the patterns of history he observes, which includes such intricacies as a discussion of relative income in the provinces of the Roman Empire in 14 AD, and a discussion of the impact of Islam on African growth. The book also finds space for a history of the art of macro-measurement, beginning with the seventeenth-century English "Political Arithmeticians."

The Maddison numbers suggest that the transition to modern growth had two phases. Before 1000 AD all societies were close to a subsistence minimum GDP per capita, which he takes as $400 (1990 international prices), and income stagnated for eons.[3] Then between 1000 and 1820 there is a period of slow but persistent economic growth in Western Europe, which tripled average real incomes there by 1820. Europe in these years was gaining decisive advantage over Asia in living standards. After 1820 there was the marked acceleration of growth rates associated with the Industrial Revolution, and its spread to the rest of the world. This discovery of a two-phase process of growth has important implications for growth theorists trying to model long-run growth.

There is, however, a problem at the core of the book, and indeed at the core at the whole Maddison project for at least the last ten years. All the numbers Maddison estimates for the years before 1820 are fictions, as real as the relics peddled around Europe in the Middle Ages. Many of the numbers for the years 1820, 1870, and 1913

[1] Maddison, *Economic Growth*.

[2] It actually revised the estimates provided earlier by Maddison, *World Economy: A Millenial Perspective.*

[3] Except, as noted, curiously Nepal, with its $397 GDP per capita.

are equally fictive. Just as in the Middle Ages, there was a ready market for holy relics to lend prestige to the cathedrals and shrines of Europe—Charlemagne secured for the cathedral in Aachen, his capital, the cloak of the Blessed Virgin, and the swaddling cloths of the infant Jesus—so among modern economists there is a hunger by the credulous for numbers, any numbers however dubious their provenance, to lend support to the model of the moment. Maddison supplies that market.

How exactly Maddison created his numbers for many countries and epochs is never precisely clear. One crucial element is his assumption that the basic subsistence GDP per capita of all societies is $400 (1990 international prices). This is the fundamental constant in Maddison's world, the basic unit of human existence. Any society without a sophisticated production technology, without significant urbanization, and without a substantial rich class, or just where nothing is known, is assigned this minimum. Thus around 1000 AD the various parts of the world are mostly assumed to have incomes either of $400 (uncivilized) or $450 (civilized). Of 27 quotes of income per capita for 1000 AD for individual countries or regions, 26 lie between $400 and $450 (p. 382). Those where our ignorance is largely complete all get assigned this $400: United Kingdom, 1 AD, $400, United Kingdom, 1000 AD, $400, and Mexico, 1 AD, $400.

Why $400 is the assumed subsistence income is not explained. Maddison has no estimates from 1820 on, where data does exist, for income per person for the types of societies this number is supposed to apply to. The only societies reported to have close to these income levels in 1820 are those of Africa (and Nepal). But they have such incomes not because Maddison has data on their GDP. There is no such data for 1820. It is because in the absence of such data, he assumed that they lay close to his subsistence primitive.

Yet this subsistence assumption is vital to his whole account of the development of the world economy before 1820. Since by 1820, when we get closer to real data, almost all societies are found to have incomes well in excess of this, inevitably we have economic growth between 1000 and 1820. Had Maddison assumed subsistence was $700, there would be no growth from 1 AD to 1820.

What is that subsistence income in real terms? In 1990 US $ prices, a pound of white bread cost $0.70. So Maddison's $400 is the equivalent of 1.6 lbs of wheaten bread per person per day, or 1,500 kcal.[4] That is an extraordinarily low income, rarely observed in practice. Since most societies have inequality, the poorest in such a subsistence economy would have lived on the equivalent of much less than that daily 1.6 lbs of bread. So if the poorest people spent nothing on clothing, heat, shelter, light, and consumed only the cheapest form of calories such as bread, they would still be engaging in hard physical labor on a diet well below 1,500 kcal in the Maddison vision of subsistence.

There is ample evidence, however, that incomes even of the most "primitive" societies greatly exceeded the Maddison assumption. Modern anthropologists, for example, have recorded the daily food consumption of surviving hunter-gatherer and shifting cultivation groups. The median consumption per person was 2,340 kcal per day, well above Maddison's assumed subsistence.[5] Many of these calories came as much more expensive meat. So just measuring the value of their food consumption, hunter-gatherers, the most primitive of the primitives, lived at an income equivalent to at least double Maddison's $400.

[4] This is confirmed from historical data. Maddison estimates a U.K. GDP per capita of $1,706 in 1820. U.K. incomes then supplied the inhabitants with the daily equivalent of 6.4 lbs of wheaten bread, implying 1 lb of bread was equivalent to $0.73.

[5] Clark, *Farewell to Alms*, table 3.6, p. 50.

Human heights supply further evidence on early living standards. Heights increase with income, which increases the quantity and quality of foods. Thus the average English male around 1820, when income per capita on Maddison's measure would be $1,900, was only about 168 cm tall (66 inches), compared to 70 inches for the richest modern societies. In contrast for Indians around 1820, where Maddison reports an income of $533, average male heights were only 162 cm (64 inches).[6] What were the heights then of people supposedly living on $400 a day, who should be smaller even than the Indians and Chinese in the nineteenth century? For modern hunter-gatherers and shifting cultivators, the median is 165 cm. For Mesolithic and Neolithic Europeans, as evidenced by skeletons, it was 169 cm, taller even than the rich English of 1820. For a variety of societies observed for 1000 AD and before, when in Maddison's vision all societies hewed close to the starvation minimum, the median was 166 cm, little less than the prosperous English of 1820.[7]

For the years 1250–1820 there is also ample evidence of real wages across a variety of countries. These wages have been collected in recent years by a whole variety of economic historians: Robert Allen, Jean-Pascal Bassino, Giovanni Federico, Debin Ma, Paolo Malanima, Sevket Pamuk, and Jan Luiten van Zanden. There are also scattered sources on wages in various early localities. Wage payments are typically 50–75 percent of total income in societies. Thus these wage rates can be used to set a lower bound on real GDP in earlier societies.

The unskilled wage of preindustrial workers before 1800 is generally far above Maddisonian subsistence. Assume 300 days of work per year, 40 percent of the population working, all wages at the unskilled level, and the wage share in national income as high as 70 percent. Then a society with a GDP per capita of $400 would have an unskilled day wage of 3.4 lbs of bread. In contrast, the day wages of farm laborers in England in the 1440s were the equivalent of 20 lbs of bread per day, about six times Maddison's subsistence. For the earliest year we have evidence for England, 1209, the implied unskilled day wage was still the equivalent of 15 lbs of bread.[8] There are only a few societies that ever report real unskilled wages possibly consistent with Maddison's subsistence assumption.

Maddison, conscious of the difficulty of reconciling his assumptions about economic growth between 1 AD and 1820 with this copious preindustrial wage information, simply rejects it all as "primitive" and "almost universally rejected as a proxy for GDP per capita."[9] Instead he prefers to feel his way, ad hoc, between his GDP estimates for each society from whenever there is actual output data, and the time in the past when GDP was $400, using estimates where they exist of urbanization, or the labor share in agriculture. Thus for Britain, for example, Maddison just assumes that that the growth rate of GDP per person in 1500 to 1700 was the same as estimated by Nick Crafts and Knick Harley for 1700–1801, which interval of course includes part of the Industrial Revolution.[10] For other European countries, Maddison imposes ad hoc growth rates between 1500 and 1820. Austria, Denmark, Finland, Sweden, and Switzerland have exactly the same rate of growth of GDP per person from 1500 to 1820, 0.170 percent precisely.

For Italy, Maddison lists a GDP per capita of $1,100 for 1500, 1600, 1700, and $1,117 in 1820. Maddison presumably preferred to believe that Italian GDP per capita did not decline between the Renaissance and 1820 because Italian urbanization

[6] Ibid., table 3.8, p. 57.
[7] Ibid., tables 3.9, 3.10, pp. 59–61.
[8] Clark, "Farm Wages."
[9] Maddison, *World Economy: Historical Statistics*, p. 253.
[10] Maddison, *World Economy: A Millennial Perspective,* p. 246.

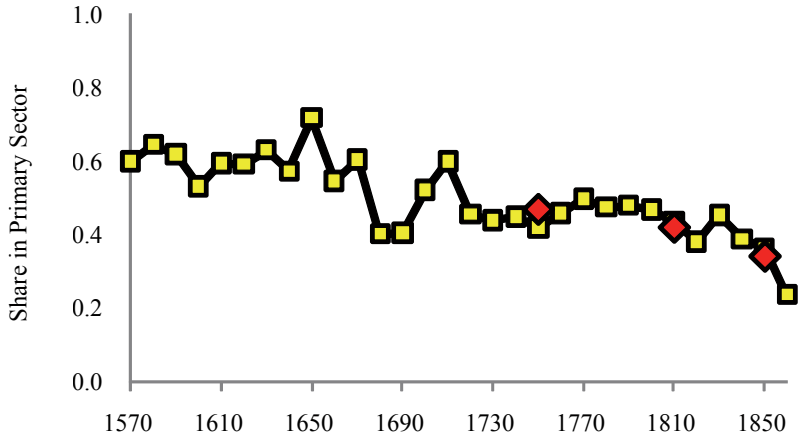

FIGURE 1
THE SHARE OF PRIMARY SECTOR EMPLOYMENT IN ENGLAND FROM WILLS, 1570–1869

Notes: The diamonds show the estimates of Shaw-Taylor and Wrigley. The wide swings in shares 1650–1739 are caused by small sample sizes in these years.

rates changed little over this interval, being around 14 percent throughout.[11] Federico and Malanima suggest, however, that real wages in north and central Italy fell by nearly 50 percent between 1500 and 1800.[12]

Urbanization is used as an indicator of per capita GDP since it is presumed to be a measure of the share of the population employed outside the primary sector. For example, for England in the years 1550 to 1800 there is a well-attested rise in the urbanization rate from 3.5 percent to 20.3 percent of the population, with in the same period no gain in rural or urban real wages.[13] The presumption from the low urbanization rate in 1550 is that the share of the population employed in the primary sector must be 70–80 percent, with a consequently low implied GDP per capita. However, it is possible to estimate the share of people employed in the primary sector for England from 1570–1860 using men's wills, which often state the testator's occupation. Figure 1 shows the implied share in primary production by decade 1570s–1860s. Also shown are recent extensive estimates by Leigh Shaw-Taylor and Anthony Wrigley for 1755, 1817, and 1851. In the decades of overlap, the estimates coincide well. But the wills show that England back in 1570, with a 3.5 percent urban share, had only 60 percent of men employed in the primary sector, compared to 46 percent by 1800. The vast majority of those in secondary and tertiary occupations were located in the countryside. Urbanization in the preindustrial world consequently is not the reliable predictor of consumption and employment patterns, and hence of income, that Maddison presumes.

England 1209–1800 is probably the best documented of all preindustrial economies. Yet we see above that even in England there is still debate about how much, if any, economic growth there was between 1209 and 1800. Those who believe there

[11] Share in towns of more than 10,000 people.
[12] Clark, *Farewell to Alms*, figure 3.3, p. 47.
[13] de Vries, *European Urbanization*, p. 39.

was significant growth in the years 1300–1800 in England have been forced to reconcile this with the contrary evidence of high early real wages by positing an "Industrious Revolution," for which there is minimal direct evidence, which dramatically increased work hours per person.[14] If the path of GDP per person even in England between 1300 and 1800 is a matter of ongoing dispute, no consensus is possible on what it was in Finland, China, India, Africa, Nepal, or anywhere else. Maddison also presents detailed population estimates, and estimates of total GDP, for the years before 1820. The population estimates are largely drawn from Colin McElvedy and Richard Jones's *Atlas of World Population History*. Inspection of that source reveals that it is similarly largely a work of imagination in these earlier years. This book, for example, happily quotes estimates of the population of the Indian subcontinent for the years 1820 and earlier. Yet examination of the sources reveals there are essentially no data on population for India, at present, for the years 1820 and before. All this material is suitable for use only in the most general and qualified way.

Despite this these numbers on GDP per capita, by country, have been widely used to lend support to theoretical models. Maddison's data for the years 1820 and earlier has been cited as tests of the theories expressed in papers published in the *American Economic Review, Quarterly Journal of Economics, Journal of Monetary Economics, Journal of Economic Growth,* and this JOURNAL, among others.[15] Somehow the Maddison numbers have an imprimatur that is completely out of line with their dubious provenance. The authors using them are happily employing as "data" what is not data at all, but the expression of one man's unexamined theory of what must have happened in the last two millennia of human history.

The interpretive essays in *Contours of the World Economy* cover a vast range of history, but mainly consist of summaries of the economic history of various parts of the world, in the light of the new GDP estimates, without any particular theme or underlying model. Thus they are not a noteworthy attraction of the book. Maddison's latest set of numbers, laid out in the tables of this book, are his claim to fame. For the reasons given above, however, any economist with enough street savvy to resist fabulous riches offered by unknown Nigerians over the internet will equally want to steer clear of these estimates.

[14] de Vries, *Industrious Revolution*; and Broadberry et al., "British Economic Growth."

[15] See, for example, Acemoglu, Johnson, and Robinson, "Rise of Europe" and "Reversal of Fortune"; Boucekkine, de la Croix, and Licandro, "Early Mortality Declines"; Falkinger and Grossman, "Institutions and Development"; Mokyr, "Long-Term Economic Growth"; Ngai, "Barriers"; and Sylla, "Financial Systems."

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