University College London, 2018-2019
Econ 0039 – Advanced Macro – Franck Portier

Problem set 2: The liquidity trap
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Solution for Part I

Part I – The Krugman Liquidity Trap model with more general preferences

We consider the Krugman model studies in class (endowment economy, cash-in-advance, representative agent). The only difference is that we assume now that preferences are given by:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Budget and Cash in Advance constraints are

$$\begin{aligned} B_{t+1} + M_{t+1} + P_t c_t &\leq (1+i_t)B_t + M_t + P_t y_t + X_t && (\lambda_t) \\ P_t c_t &\leq M_t && (\mu_t) \end{aligned}$$

We consider first the case in which prices are flexible.

1 – Give the fist order conditions of the (representative) household problem, for given prices P and i, not forgetting the complementary slackness conditions.

The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^t u(c_t) \\ &+ \sum_{t=0}^{\infty} \lambda_t \left((1+i_t)B_t + M_t + P_t y_t + X_t - B_{t+1} - M_{t+1} - P_t C_t\right) \\ &+ \sum_{t=0}^{\infty} \mu_t \left(M_t - P_t C_t\right) \end{aligned}$$

and FOC are $\forall \ t \geq 0$

$$\begin{aligned} \text{w.r.t. } c_t &: & \beta^t u'(c_t) &= (\mu_t + \lambda_t) P_t \\ \text{w.r.t. } B_{t+1} &: & \lambda_t &= (1+i_{t+1})\lambda_{t+1} \\ \text{w.r.t. } M_{t+1} &: & \lambda_t &= \lambda_{t+1} + \mu_{t+1} \\ \text{slackness} &: & \mu_t(M_t - P_t c_t) &= 0 \\ & & \mu_t &\geq 0 \end{aligned}$$

2 – Prove that λ is always positive.

Assume $\lambda_t = 0$. Then the third FOC implies $\lambda_{t+1} + \mu_{t+1} = 0$. As multipliers are positive or nul, this implies $\lambda_{t+1} = \mu_{t+1} = 0$. Therefore, the first FOC implies $\beta^t u'(c_t) = 0$. This is not possible because $u'(c_t) = c_t^{-\sigma} > 0$. Therefore $\lambda_t = 0$ is not possible, so that $\lambda_t > 0$ for all t.

3 – Whey do we assume $i_t \geq 0$?

It is a technological constraint: one cannot reduce the nominal value of a coin or a note that is in an economic agent pocket.

4 – What are the three market equilibrium conditions?

All agents are identical (representative household) $\rightsquigarrow$ The equilibrium on the bond market is

$$B_t = 0$$

Money market equilibrium:

$$M_{t+1} = M_t + X_t$$

Good market equilibrium

$$c_t = y_t$$

5 – Assume that endowments and money supply are constant in period 1 and onwards, solve for equilibrium for 1 and onwards.

$c_t = y_t = y^\star$ implies

$$1 + r^\star = \frac{1}{\beta} > 1$$

such that $r^\star > 0$

Let us guess an equilibrium in which the CIA constraint binds:

$$P^\star = \frac{M^\star}{y^\star}$$

In such a case,

$$1 + i_t = \frac{(1 + r_t)P_{t+1}}{P_t}$$

implies

$$1 + i^\star = \frac{(1 + r^\star)P^\star}{P^\star} = 1 + r^\star$$

Therefore $i^\star = r^\star > 0$ Because $i^\star > 0$, the CIA binds, so that our guess was correct.

6 – Show that the cash-in-advance constraint might or might not bind in period zero. What is the condition under which it is binding?

The equilibrium in period $t \geq 1$ is therefore $y^\star$, $i^\star = r^\star = 1/\beta$, $P^\star = M^\star/y^\star$

The equilibrium (i, P) in period 0 is given by the equations:

$$\begin{aligned} i &= \frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma \frac{P^\star}{P} - 1 \\ P &= \frac{M}{y} \quad \text{if } i > 0 \\ P &= \overline{P} \quad \text{if } i = 0 \end{aligned}$$

where $\overline{P}$ is defined by $\frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma \frac{P^\star}{\overline{P}} - 1 = 0$. The cash-in -advance constant is binding if $i > 0$, meaning $P < \overline{P}$. Given, $\frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma \frac{P^\star}{\overline{P}} = 1$ with $\overline{P} = \overline{M}/y$, we have that the CIA constraint is binding if $M < \overline{M}$ with $\overline{M} = \frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma P^\star y$.

Note that $\frac{P^\star}{P} - 1$ represents expected inflation

7 – Assume that prices are sticky in period 0, such that consumption c can be lower than endowment y. Under which condition is there a liquidity trap and under-consumption $(c < y)$?

See the slides of lecture 9. This will happen if the level of c for which $i = 0$ is below y. That level of c, $\widehat{c}$ is given by $i = 0$, which is equivalent to $\frac{1}{\beta}\left(\frac{y^\star}{\widehat{c}}\right)^\sigma \frac{P^\star}{P} - 1 = 0$, so that $\widehat{c} = \frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma \frac{P^\star}{P}$. therefore, the economy will be in the liquidity trap if the fixed price P is such that $\widehat{c} < y$, that is $P > \frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma P^\star$.

8 – How can the commitment to increase money supply in period 1 and onwards can restore efficiency of monetary policy in period 0?

By committing to increase $M^\star$, the monetary authority increases $P^\star$. It can do it such that $P < \frac{1}{\beta}\left(\frac{y^\star}{y}\right)^\sigma P^\star$. Then, the economy is out of the liquidity trap, and increasing M increases c.