University College London, 2018-2019
Econ 0039 – Advanced Macro – Franck Portier

Problem set 4: Sustainable growth
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Solution for Problems I and III

Problem I – Natural resources in the Solow model.

Some factors of production are available in finite supply, typically natural resources (land, oil, etc...). Therefore, it is often argued that economic growth should eventually stop. Let us consider this view in a Solow model.

Let the production function be $Y = K^{\alpha}(AL)^{\beta}T^{1-\alpha-\beta}$ where K is capital, L labor, T land and A productivity (think of any variable X as being $X(t)$). Assume $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. Factors evoke according to:

$$\begin{aligned} \dot{K} &= sY - \delta K \\ \dot{A} &= gA \\ \dot{L} &= nL \\ \dot{T} &= 0 \end{aligned}$$

1 – Discuss the model assumptions

Those are the standard assumptions of the Solow model, except for land T, which is in fixed supply. Notice that it is not different from the case of labor in the Solow model if population growyh is zero.

2 – Does the economy have a unique and stable balanced growth path, where all variables grow at a constant but not necessarily similar rate)? If so, what are the growth rates? If not, why not?

We have $\dot{A}/A = g$,$\dot{L}/L = n$ and $\dot{T}/T = 0$. We need to show that $\dot{Y}/Y$ and $\dot{K}/K$ also converge to a constant. The equation for capital accumulation is

$$\dot{K} = sY - \delta K$$

Dividing by K gives:

$$g_K = \frac{K}{K} = s\frac{K}{Y} - \delta$$

This equation tells us that Y and K need to grow at the same rate if the growth rate of capital is to be constant along a BGP. Therefore, $g_K = \dot{Y}/Y$ along a BGP. Next, take the time derivative of the production technology and replace for the growth rates:

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \beta\frac{(\dot{AL})}{AL} + (1-\alpha-\beta)\frac{\dot{T}'}{T}$$

which writes

$$g_K = \alpha g_K + \beta(g+n)$$

so that

$$g_K = \frac{\beta}{1-\alpha}(n+g)$$

The economy has a BGP in which the growth rate of capital and output is constant and equal to $(n+g)$, and productivity and labor grow at exogenous constant rates g and n respectively.

3 – Given your answer to the previous question, does the fact that the stock of land is constant imply that permanent growth rate is not possible? Explain.

4 – Under which circumstance will $\frac{Y}{L}$ grow over time, as opposed to shrinking steadily? Give your answer in terms of α, β, n and g.

This is an answer to the two previous questions. Permanent growth is possible with a fixed stock of land. The reason is that even though land is constant, the number of effective workers is growing at exogenous constant rate. Let's compute the growth rate of putput per capital along a BGP:

$$\frac{(\dot{Y/L})}{Y/L} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = g_K - n = \frac{\beta}{1-\alpha}(n+g) - n = \frac{\beta g - (1-\alpha-\beta)n}{1-\alpha}$$

As we can see from this equation, the growth rate of output per capita can be either positive or negative. Two observations should be made. First, the bigger the rate of technological progress relative to population growth, the less likely it is that output per capita decreases. Second, the bigger the coefficient on land in the production function

$(1-\alpha-\beta)$ compared to the one on technological progress β, the more likely it is that output per capita growth is negative. Decreasing returns on the fixed factor (land) are dragging down the growth of income per capita.

5 − The Malthusian view assesses that if resource are finite, then the economy will end in a no growth trap. Discuss that prediction in the light of this model.

There are two differences between this model and the mechanism Malthus had in mind. The first is that he did not consider the rate of population growth as constant, but as endogenous, being influenced by the level of output per capita. The second difference is that he did not think about technological progress. As we have shown above, technological progress makes a difference: if technical progress is fast enough, it may offset the decreasing returns effect generated by the fixed stock of land.

Suppose the growth rate of a population of wild fish is given by $\dot{x} = F(x) = 10x - 0.01x^2$ if the population is left undisturbed, were x is population size.

1 – What is the equilibrium natural population?

The equilibrium population level occurs when the growth rate of population is zero and the population is positive. That is, $10x = 0.01x^2$ and $10 = 0.01x$. Thus, $x = 1000$ fish

2 – What is the maximum sustained yield, and the corresponding population? Note that the maximum sustained yield is the maximum number of fish that can be caught per period while keeping the population of fish constant.

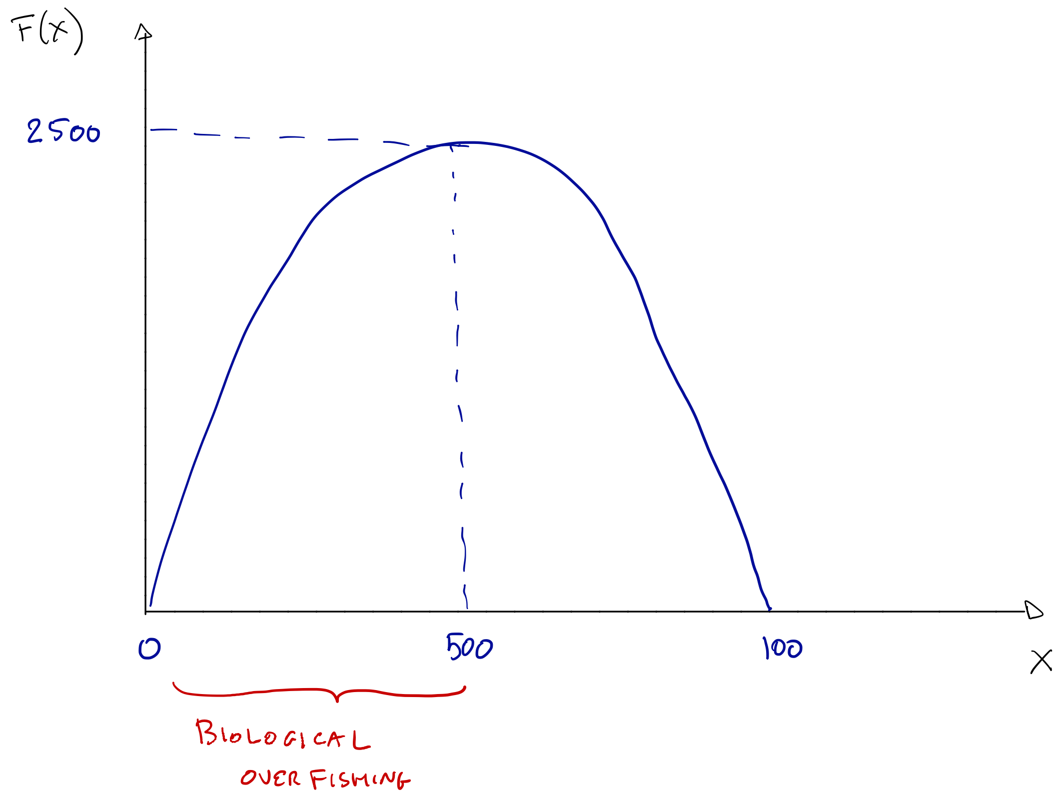
The maximum sustained yield occurs at the maximum growth rate. The FOC is

$$dF/dx = 10 - 0.02x = 0,$$

which implies that $x = 500$ at MSY.
The Second Order Condition is $F" = -0.02 < 0$, so $x = 500$ is a maximum.
The growth rate at MSY is $10 \times 500 - 0.01 \times 500^2 = F(500) = 2500$ fish/unit of time. It will be possible to capture 2500 fish per unit of time while keeping the fishery population at 500 and then to capture another 2500 per unit of time indefinitely. So MSY = 2500 fish.

3 – Graph the population growth function. Which portions of this function correspond to biological overfishing (meaning that the population will tend to zero because the number of catch)?

See graph below. Biological overfishing corresponds to all populations less than (left of) $x = 500$.



Suppose now that the number of fish caught (harvest) is given by $h = 2Ex$, where E is the amount of fishing effort.

4 – Find the sustainable population x as a function of effort E.

Now the net growth rate of fish is

$$F(x) - h = 10x - 0.01x^2 - 2Ex.$$

The equilibrium population level occurs when the net growth rate of population is zero and the population is positive. That is, $(10 - 2E)x = 0.01x^2$ and $10 - 2E = 0.01x$. Thus, $x = 1000 - 200E$.

5 – Find sustainable yield as a function of E.

We can compute the yield-effort curve which will take the form

$$Y = qKE(1 - qE/r),$$

with $r = 10$, $q = 2$, $K = 1000$. Thus,

$$Y = 2000E\left(1 - \frac{E}{5}\right) \Leftrightarrow Y = 2000E - 400E^2$$

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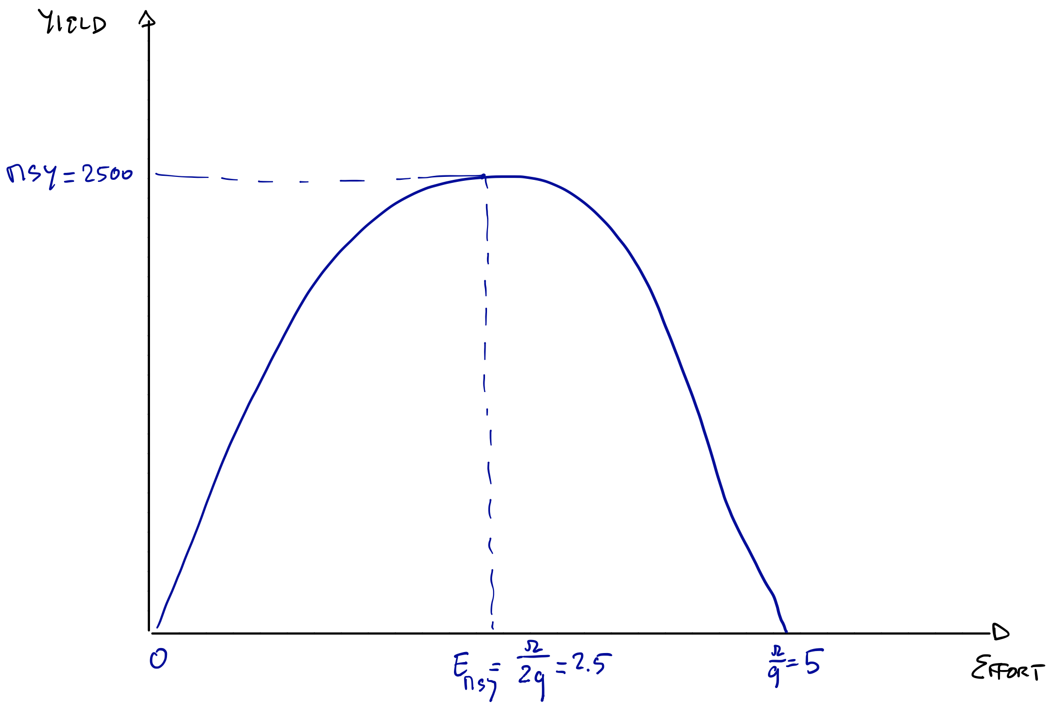
6 – Graph the resulting Yield-Effort curve. Which portions of this curve correspond to biological overfishing?

See graph below. Biological overfishing corresponds to all levels of effort to the right of E_{MSY}.



Suppose now that the price of fish is $P = 10$, and total cost of fishing effort is $C(E) = 1000E$.

7 – Find the Total Revenue Product Curve.

TRP $= P \times Y(E)$. As $Y(E) = 2000E - 400E^2$, we have

$$\text{TRP} = 20000E - 4000E^2.$$

8 – Suppose the fishery is managed by an owner with exclusive property rights. Find the level of effort E that maximizes the rent (revenue) from the fishery, and calculate that maximum rent.

The rent-maximizing level of effort will be the level of effort that maximizes the following function:

$$\text{Rent} = TRP - TC(E) = 20000E - 4000E2 - 1000E = 19000E - 4000E^2.$$

FOC is

$$\frac{d[\text{Rent}]}{dE} = 19000 - 8000E = 0$$

so that $E = 19/8 = 2.375$. Maximum Rent will therefore be $19000 \times 2.375 - 4000 \times 2.3752$, so that Rent = 22,562.50.

9 – Under open access to the resource, any fisher can access the resource, so that the only equilibrium level of fishing is the one that drives the rent to zero. Find the level of effort that would occur under open access.

Under open access, economic rent from the fishery would diminish to zero and $19000E - 4000E^2 = 0$, so $19000E = 4000E^2$ and the non-zero level of effort for this is $E = 19000/4000 = 19/4 = 4.75$.

10 – Under open access, is the fishing activity suffering from economic overfishing (meaning that the rent is not maximum)?

Under open access, the fishery is suffering from economic overfishing, because rents are being dissipated and the fishery is contributing no value on net to the economy. With less effort, rents could exist and the economic value of the fishery could be increased.

11 – Under open access, is the fishing industry suffering from biological overfishing? Why or why not?

Under open access, the fishery is suffering from biological overfishing. Biological overfishing occurs when E $> \frac{r}{2q}$. But here, $\frac{r}{2q} = 2.5 = E$, and $4.75 > 2.5$, so the fishery is being fished at a level of effort greater than the one which corresponds to maximum sustainable yield.