University College London, 2019-2020

Econ 0107 – Macroeconomics I – Ralph Luetticke & Franck Portier

Midterm Exam

This is a 2 hours exam. No documents allowed. 100 points in total, 25 points for each problem.

I – The Welfare Cost of Business Cycles

Let utility be given by:

$$E_{-1}\sum_{t=0}^{\infty}\beta^t u(c_t)$$

where instantaneous utility takes this specification $u(c_t) = \log(c_t)$.
The consumption process is

$$c_t = c_{t-1}^{\alpha}\epsilon_t exp(\mu)$$

where $\mu = -\frac{\sigma_\epsilon^2(1-\alpha)}{2(1-\alpha^2)}$ and $\log\epsilon_t \sim iid - N(0,\sigma_\epsilon^2)$. So that the logarithm of consumption follows an AR(1) with parameter α:

$$\log c_t = \mu + \alpha\log c_{t-1} + \log\epsilon_t$$

1 – In the long run, $\log c_t$ converges (in distribution) to $N\left(\frac{\mu}{1-\alpha}, \frac{\sigma_\epsilon^2}{1-\alpha^2}\right)$, which we call the invariant distribution. Show that the normalization proposed is such that the mean consumption is equal to one.
Hint: Let Z be a standard normal variable, and let μ_Z and $\sigma_Z > 0$ be two real numbers. Then, the distribution of the random variable $X = e^{\mu_Z+\sigma_Z Z}$ is called the log-normal distribution with parameters μ_Z and σ_Z. The mean of a lognormal distribution is given by $\exp(\mu_Z + \sigma_Z^2/2)$.

2 – Assuming that consumption at time 0 (c_0) is known, find the value function

$$V(\lambda,\sigma_\epsilon^2) = E_0\sum_{t=0}^{\infty}\beta^t u(c_t(1+\lambda)),$$

where λ is a positive constant.
Hint: You can write the problem recursively as:

$$V(c;\lambda,\sigma_\epsilon^2) = u(c(1+\lambda)) + \beta EV(c';\lambda,\sigma_\epsilon^2)$$

and use a guess-and-verify approach: $V(c;\lambda,\sigma_\epsilon^2) = A\log c + B$

3 – Compute the value of λ as a function of α, σ_ϵ^2 and γ such that

$$V(c;\lambda,\sigma_\epsilon^2) = V(c;0,0).$$

What does λ represent? Determine how λ depends on α, σ_ϵ^2 and γ and interpret you results. Compare to the i.i.d. case, i.e. $\alpha = 0$.
Hint: try to use $\log(1+x)\approx x$.

4 – What happens when $\alpha\to 1$? Is it sensible to think at λ as the “welfare cost of business cycles”?

II – McCall Model

Consider a worker who draws every period a job offer to work forever at wage $w > 0$. Successive offers are independently and identically distributed drawings from a distribution $F_i(w)$, $i = 1, 2$. Assume that F_1 has been obtained from F_2 by a mean-preserving spread. The worker's objective is to maximize

$$E \sum_{t=0}^{T} \beta^t y_t, \qquad 0 < \beta < 1,$$

where $y_t = w$ if the worker has accepted employment at wage w and is zero otherwise. Assume that both distributions, F_1 and F_2, share a common upper bound, B.

1 – Write down the Bellman equation at time t. Argue that the optimal policy is of the reservation wage form.

2 – Show that the reservation wages of workers drawing from F_1 and F_2 coincide at $t = T$ and $t = T - 1$.

3 – Now introduce unemployment compensation: the worker who is unemployed collects c dollars. Show that the result in **2** – no longer holds, that is, the reservation wage of the workers that sample from F_1 is higher than the one corresponding to workers that sample from F_2 for $t = T - 1$.

III – Fiscal Policies in a Dynamic Economy

Consider the following setup. There is no uncertainty, and decision makers have perfect foresight. A representative household has preferences over nonnegative streams of a single consumption good c_t that are ordered by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad \beta \in]0, 1[$$

where u is strictly increasing in c_t, twice continuously differentiable, and strictly concave. The households supplies inelastically one unit of labor.

The technology is

$$\begin{aligned} g_t + c_t + x_t &\leq F(k_t, n_t) \\ k_{t+1} &= (1 - \delta)k_t + x_t \end{aligned}$$

where $\delta \in]0, 1[$ is a depreciation rate, k_t is the stock of physical capital, x_t is gross investment, and $F(k, n)$ is a linearly homogeneous production function with positive and decreasing marginal products of capital and labor. We have the notation $F(k, 1) = f(k)$. g stands for government expenditure.

There is a competitive equilibrium with all trades occurring at time 0. The household owns capital, makes investment decisions, and rents capital and labor to a representative production firm. The representative firm uses capital and labor to produce goods with the production function $F(k_t, n_t)$. A price system is a triple of sequences $\{q_t, r_t, w_t\}_{t=0}^{\infty}$ where q_t is the time 0 pretax price of one unit of investment or consumption at time t (x_t or c_t), r_t is the pretax price at time 0 that the household receives from the firm for renting capital at time t, and w_t is the pretax price at time 0 that the household receives for renting labor to the firm at time t.

The household faces the budget constraint:

$$\sum_{t=0}^{\infty} \Big(q_t(1 + \tau_{ct})c_t + (1 - \tau_{it})q_t\big[k_{t+1} - (1 - \delta)k_t\big] \Big) \leq \sum_{t=0}^{\infty} \Big(r_t(1 - \tau_{kt})k_t + w_t(1 - \tau_{nt})n_t - q_t\tau_{ht} \Big)$$

where τ_{ct} is the consumption tax rate, τ_{it} the investment credit rate, τ_{kt} the capital earning tax rate, τ_{nt} the labor earning tax rate and τ_{ht} a lump-sum tax.

1 – Write the government budget constraint.

2 – Define a competitive equilibrium.

3 – Use the household budget constraint to derive a no-arbitrage condition, from which you will get a formula for the user cost of capital. Comment.

4 – Write down the household maximization problem and derive first order conditions.

5 – Write down the firm maximization problem and derive first order conditions.

6 – Show that the equilibrium can be characterized by a second order nonlinear difference equation in k_t.

7 – Derive a formula for the steady state level of capital.

8 – Which taxes are distortionary at the steady state?

IV – Overlapping genearations

Consider an economy consisting of overlapping generations of two-period-lived agents. There is a constant population of N young agents born at each date $t \geq 1$. There is a single consumption good that is not storable. Each agent born in $t \geq 1$ is endowed with w_1 units of the consumption good when young and with w_2 units when old, where $0 < w_2 < w_1$. Each agent born at t$\geq$ 1 has identical preferences $\ln c_t^h(t) + \ln c_t^h(t+1)$, where $c_t^h(s)$ is time s consumption of agent h born at time t. In addition, at time 1, there are alive N old people who are endowed with $H(0)$ units of unbacked paper currency and who want to maximize their consumption of the time 1 good.

A government attempts to finance a constant level of government purchases $G(t) = G \geq 0$ for $t \geq 1$ by printing new base money. The government's budget constraint is

$$G = (H(t) - H(t-1))/p(t),$$

where $p(t)$ is the price level at t, and $H(t)$ is the stock of currency carried over from t to $(t+1)$ by agents born in t. Let $g = G/N$ be government purchases per young person. Assume that purchases $G(t)$ yield no utility to private agents.

1 – Denote by $s_t^h(t)$ the savings of young agent h, and let $r(t)$ be the rate of return on savings between t and $(t+1)$. Write down the maximization problem of agent h, and derive saving function $f^h(1 + r(t))$.

2 – Let's assume that savings can be done using currency or by issuing/subscribing privately issued bonds. Explain why the following equation holds in equilibrium when fiat currency (money) is valued:

$$1 + r(t) = \frac{p(t)}{p(t+1)}.$$

3 – Define a competitive equilibrium with valued fiat currency.

4 – Find a stationary equilibrium with valued fiat currency.

5 – Prove that for small enough g, there exist two stationary equilibria with valued fiat currency. (*You can do that graphically by using the function f to draw seigniorage revenues as a function of* $(1 + r)$).