University College London, 2020-2021
Econ 0107 – Macroeconomics I – Ralph Luetticke & Franck Portier

Midterm Coursework Assessment

Each part carries 25% of the total mark

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I – The End of the World in an OLG Model

This problem explores the consequences of the world having an end (possibly stochastically) in an OLG model. Notations will be, as much as possible, those of the course. Agents are indexed by $i = 0, 1, ..., +\infty$, where i is the period of birth. The economy starts in period 1. Each generation lives for two periods, and preferences are

$$U^i(c^i) = \log(c_i^i) + \log(c_{i+1}^i).$$

Utility is $U^0(c^0) = \log(c_1^0)$ for the old of period 0 and $\log(c_i^i)$ for the young in the last period of the world. Endowments are $(y_i^i, y_{i+1}^i) = (1-\varepsilon, \varepsilon)$ and $y_t^i = 0$ if $t \neq i, i+1$, with $\varepsilon \in]0, 1/2[$.

The part that is different from the course is that, in every period i, Nature is flipping a coin and the world will end at the end of the next period $(i+1)$ with probability π_i and will survive the end of the next period with probability $1 - \pi_i$.

In period i, the economy can be in three different states:

- $s_i = E$ if the world is ending at the end of $i + 1$;
- $s_i = \overline{E}$ if the world does not end at the end of $i + 1$;
- $s_i = \varnothing$ if period i is the last period of the world (meaning that $s_{i-1} = E$).

We always assume that $\pi_1 = 0$.

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Assume first that $\pi_1 = 0$ and $\pi_2 = 1$. In words, the economy is deterministic, and ends for sure in period 3.

1 – Assume time-0 trading and no money. Write budget constraints of agents of generations 0 to 3 (use prices q_i^0). Compute their offer curves. Show that autarky is an equilibrium and that the perfect smoothing allocation

$$\left(c_1^0, \left\{c_i^i, c_{i+1}^i\right\}_{i=1,2}, c_3^3\right) = \left(\frac{1}{2}, \left\{\frac{1}{2}, \frac{1}{2}\right\}, \frac{1}{2}\right)$$

is *not* and equilibrium. Discuss.

2 – Assume now sequential trading. A quantity of money M is distributed to the old of period 1. Denote p_i the price of one unit of good in terms of the currency. Show that there cannot be any monetary equilibrium.

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Now assume that the world never ends, –*i.e.* $\pi_i = 0\ \forall i \geq 1$.

3 – What are the two stationary equilibria with time-0 trading (and no money)?

4 – What is the unique stationary monetary equilibrium with sequential trading?

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Enough for the warm-up. Assume now that $\pi_i = \pi \in]0,1[\forall i \geq 2$. For an active period i (by active, we mean not after the end of the world), denote $x_i(s_i)$ any variable of period i, with $s_i \in \{E.\overline{E}, \varnothing\}$.

5 – Assume time-0 trading and no money. Write the maximization problem of generation $i > 1$ (expected utility and intertemporal budget constraint). Keep in mind that prices depend on the state of the world. Derive first order conditions. Can autarky be an equilibrium? Can perfect smoothing be an equilibrium? Discuss.

6 – Assume now sequential trading and that the old of generation is endowed with M units of money. Can perfect smoothing be an equilibrium? Find a condition on π for the existence of a stationary monetary equilibrium. What are the equilibrium quantities at this equilibrium. Discuss.

II – Business Cycles Movements at a Temporary Equilibrium

Data show that consumption, investment and labour all move in the same direction in the business cycle. This is what we call "business cycle movements". We are here interested in how the economy responds to a news about a future change in the economy, that does not affect current preferences nor technology. Can this create a business cycle movement?

The model we consider is a perfect competition complete market growth model. Let's assume the economy is deterministic. Preferences are

$$\sum_{t=0}^{\infty} \beta^t \Big(U(c_t) - V(L_t)\Big)$$

and technology is

$$Y_t = F(K_t, L_t)$$

and

$$K_{t+1} = (1-\delta)K_t + I_t.$$

It is assumed that $V_L > 0$, $V_{LL} > 0$, $U_C > 0$, $U_{CC} < 0$, $F_K > 0$, $F_{KK} < 0$, $F_L > 0$, $F_{LL} < 0$ and $F_{KL} > 0$. We also assume that F is concave. K_0 is given and the transversatity condition

$$\lim_{T\to\infty} \beta^T U_{C_T} K_{T+1} = 0$$

is imposed.

1 – Write the Planner problem (that will give competitive equilibrium quantities) and derive, for each $t > 0$, three equations that can be interpreted as a good market equilibrium (GM) condition, a labour market equilibrium condition (LM) and a Euler equation.

2 – Temporary equilibria are defined as the triplets (C_t, L_t, I_t) that satisfy (GM) and (LM). Why is that an interesting concept?

3 – Assume we are in period t and that an unexpected change in future U or F occurs (a "news"). Explain why such a news will directly enter only in the Euler equation, so that in period t, the economy will have to move along (GM) and (LM).

4 – Fully differentiate (GM) and (LM) (note that K_t is predetermined and therefore cannot change) and, for a given I_t, draw those two equations in the plane (C_t, L_t).

5 – Assume that the news causes an increase in I_t ($dI_t > 0$). Show that then consumption and labour will move in opposite direction. Can news create business cycle movements in that economy?

6 – Assume now that technology is given by $C = G(K, L, I)$, with $G_K > 0$, $G_{KK} < 0$, $G_L > 0$, $G_{LL} < 0$, $G_{KL} > 0$ and $G_I < 0$. Show that this formulation encompasses the previous case.

7 – Show that temporary equilibria can now display business cycle fluctuations following a news shock if G_{LI} is positive and large enough. How would you interpret that?

III – Incomplete Markets and Portfolio Choice

Consider a model with a continuum of households who are subject to idiosyncratic returns on their investment. In particular, each household can invest in a risk-free bond with rate of return r and a risky capital whose rate of return is random and variable across individuals. Assume that households can borrow using the risk-free bond up to the natural borrowing limit while their investment in risky capital has to be positive. More specifically, the household problem reads:

$$\max_{c_t,\phi_t} \quad \sum_{t=0}^{\infty} \beta^t \ln c$$
$$s.t. \quad A_{t+1} = (A_t - c_t)((1+r)\phi_t + (1+z_t)(1-\phi_t)) + e,$$

where A is cash-on-hand and e is a constant endowment. The household decides on consumption, c_t, and the fraction of savings, ϕ_t, invested in the risk-free asset. The fraction $(1-\phi_t)$ of savings goes into capital with stochastic return z_t (iid across time and households).

1 – What is the natural borrowing limit for the risk-free asset?

2 – Write down the Bellman equation corresponding to the household problem. What is the state variable?

3 – Derive the first-order conditions. What is the interpretation?

4 – Guess and verify the solution to the Bellman equation. (Hint: ln utility!) Describe the properties of the optimal consumption and portfolio policy.

5 – Discuss the implications for aggregate asset holdings. Under what conditions does it converge?

6 – Now suppose that we have a closed economy consisted of individuals like above who are symmetric and can trade the risk-free bond. Define an equilibrium in this economy.

7 – Now suppose you would like to analyse aggregate shocks, e.g. a shock to the variance of capital returns. What state variables are needed to describe the evolution of the economy? How would you compute the solution?

IV – McCall Model

Consider a worker who draws every period a job offer from a cumulative distribution $F(w)$ to work forever at wage w. Each time the worker draws a job offer a cost C incurs, where $0 < C < E[w]$. The worker's objective is to maximize the expected value of $w - nC$, where w is the accepted wage offer and n is the number of job offers the worker has drawn. Let V denote the expected value of $w - n'C$ of a worker who has just rejected a job, where n' is the number of jobs the worker will draw from that point on.

1 – Explain why the worker accepts a job offering $\tilde{w}$ if $\tilde{w} > V$, and rejects its if $\tilde{w}$ if $\tilde{w} < V$.

2 – Explain why V satisfies $V = F(V)V + \int_{w=V}^{\infty} wf(w)dw - C$, where f is density function associated with F.

3 – Does a searcher ever want to accept a job that has been previously rejected?

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Now, suppose w is distributed uniformly on $[\mu - a, \mu + a]$ and that $C < \mu$. This implies that $f(w) = 1/2a$ and $F(w) = \frac{w-(\mu-a)}{2a}$.

4 – Find V in terms of μ, a, and C.

5 – How does an increase in a affect V? Explain intuitively.