UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**ECON0107**
ASSESSMENT PATTERN	:	**Mid Term – 24 Hour Online**
MODULE NAME	:	**Macroeconomics**
LEVEL:	:	**Postgraduate**
DATE	:	**Wed 12th Jan 2022**
TIME	:	**12:00**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

Year
2021/22

Additional material	
Special instructions	
Exam paper word count	

TURN OVER

University College London, 2021-2022
Econ 0107 – Macroeconomics I – Ralph Luetticke & Franck Portier

Midterm Coursework Assessment

Problem I carries 50% of the total mark, Problem II 40% and Problem III 10%.
Students are expected to spend 4 hours on the paper.

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I – A Model with Heterogenous Agents

Assume there is a continuum of households, each have quadratic felicity in consumption $u(c) = \alpha_1 c - \alpha_2 c^2/2$. Households maximize the discounted stream of felicity, discounted by factor β. Each household faces stochastic income y, but the aggregate income is constant. There is no liquidity constraint and households can save and dissave in a riskless bond a at return R (with a No-Ponzi condition). The budget constraint reads: $a_{t+1} = R(a_t + y_t - c_t)$.

A. Household Behavior

1 – Set up the household problem and derive the Euler equation. Argue that consumption is a martingale, if the interest rate always equals the time-preference rate.

See lecture for solution.

2 – Derive the intertemporal or net present value budget constraint and use it to derive the consumption function (assume that the interest rate always equals the time- preference rate for questions 2, 3 and 4).

See lecture for solution.

3 – Show that the change in consumption, $c_{t+1} - c_t$, is related to the news the household receives about future income. Show that consumption is a martingale using this condition.

Take the difference of the consumption function and substitute the budget constraint to obtain: $c_{t+1} - c_t = (R-1)\sum_{s=t+1}^{\infty} R^{-(s-t)}(E_{t+1}y_s - E_t y_s)$. Take expectations as of time t on both sides to recover $E_t c_{t+1} - c_t = 0$.

4 – Now assume that income follows an autoregressive process: $y_{t+1} = \rho y_t + \eta_{t+1}; \quad 0 < \rho \leq 1$. Derive an expression for the change in consumption and how it depends on η_{t+1}. Is consumption a martingale? Is it feasible to test this in data? What is the disadvantage relative to Hall (1978)'s approach.

Plug income process, $E_t y_s = \rho^{s-t} y_t$, into consumption function and rearrange: $c_{t+1} - c_t = ((1-\beta)/(1-\rho\beta))\eta_{t+1}$. Since $\rho \leq 1$, $c_{t+1} - c_t \leq \eta_{t+1}$, that is, consumption in general responds less than 1 for 1 to a change in income. The case where consumption moves 1 for 1 is when $\rho = 1$, i.e. income itself is a random walk.
The innovation to consumption ϵ_{t+1} and the innovation to income η_{t+1} are linked in a very precise way: $\epsilon_{t+1} = ((1-\beta)/(1-\rho\beta))\eta_{t+1}$. Both can be directly observed in the data and hence this equation could be estimated. However, this could be exploited only if the household learns about the change in its income as it happens. If instead, the household learns about a change in its income before it is realized, this is when consumption will change, and not when the actual change in income occurs. Unless the econometrician has information on when the information becomes available to the household (more on this below), then the relationship above will not be terribly useful. Testing the first order condition as in Hall(1978) remains valid, however, since any information known at time t to the household should not help predict future consumption.

B. Equilibrium

5 – Argue that the time preference rate is the equilibrium interest rate in this economy, when the bond market needs to clear at aggregate holdings $\int a = 0$.

The consumption function, derived under the assumption that the interest rate always equals the time-preference rate, allows for aggregation and yields that aggregate consumption equals the constant aggregate endowment. This verifies the guess. Hence, we can think of a representative consumer and apply complete markets reasoning.

6 – Suppose all income shocks are i.i.d. Is the equilibrium in question 5 a stationary one?

No. Aggregate endowment and interest rate are constant, but consumption and assets follow a random walk and hence the distribution does not converge to a stationary one.

7 – Now assume that there is an aggregate endowment increase in one specific period (known in advance). Will that affect aggregate consumption in other periods in equilibrium? How can it affect individual consumption (it suffices to extrapolate, no maths!)?

Aggregate consumption in other periods is not affected because it's equal to the fixed endowment in each period. Individual consumption is differentially affected depending on the composition of permanent income in terms of human capital y and assets a. Given the higher endowment in the future, households would like to move consumption forward and decrease savings. This is not possible as aggregate consumption is equal to the unchanged endowment and hence the interest rate increases to restore savings. A higher interest rate benefits households with relatively more wealth in assets a and lowers the permanent income of households with relatively more human capital y. Hence, consumption of these households responds differently.

C. Measurement

The above model implies: $c_t = c_{t-1} + e_t$, where e is white noise. Assume that we only observe average consumption over two-period intervals in the data; that is, $(c_t + c_{t+1})/2, (c_{t+2} + c_{t+3})/2$, and so on.

8 – Find an expression for the change in measured consumption from one two-period interval to the next in terms of the e's.

Solution: See print on next page.

9 – Is the change in measured consumption uncorrelated with the previous value of the change in measured consumption? In light of this, is measured consumption a random walk?

Solution: See print on next page.

10 – Given your result in question 1, is the change in consumption from one two- period interval to the next necessarily uncorrelated with anything known as of the first of these two-period intervals? Is it necessarily uncorrelated with anything known as of the two-period interval immediately preceding the first of the two-period intervals?

Solution: See print on next page.

(a) We need to find an expression for $[(C_{t+2} + C_{t+3})/2] - [(C_t + C_{t+1})/2]$. We can write C_{t+1}, C_{t+2} and C_{t+3} in terms of C_t and the e's. Specifically, we can write

(1) $C_{t+1} = C_t + e_{t+1}$,

(2) $C_{t+2} = C_{t+1} + e_{t+2} = C_t + e_{t+1} + e_{t+2}$, and

(3) $C_{t+3} = C_{t+2} + e_{t+3} = C_t + e_{t+1} + e_{t+2} + e_{t+3}$,

where we have used equation (1) in deriving (2) and equation (2) in deriving (3). Using equations (1) through (3), the change in measured consumption from one two-period interval to the next is

(4) $$\frac{C_{t+2} + C_{t+3}}{2} - \frac{C_t + C_{t+1}}{2} = \frac{(C_t + e_{t+1} + e_{t+2}) + (C_t + e_{t+1} + e_{t+2} + e_{t+3})}{2} - \frac{C_t + (C_t + e_{t+1})}{2},$$

which simplifies to

(5) $$\frac{C_{t+2} + C_{t+3}}{2} - \frac{C_t + C_{t+1}}{2} = \frac{e_{t+3} + 2e_{t+2} + e_{t+1}}{2}.$$

(b) Through similar manipulations as in part (a), the previous value of the change in measured consumption would be

(6) $$\frac{C_t + C_{t+1}}{2} - \frac{C_{t-2} + C_{t-1}}{2} = \frac{e_{t+1} + 2e_t + e_{t-1}}{2}.$$

Using equations (5) and (6), the covariance between successive changes in measured consumption is

(7) $$\text{cov}\left[\left(\frac{C_{t+2} + C_{t+3}}{2} - \frac{C_t + C_{t+1}}{2}\right), \left(\frac{C_t + C_{t+1}}{2} - \frac{C_{t-2} + C_{t-1}}{2}\right)\right] =$$

$$\text{cov}\left[\left(\frac{e_{t+3} + 2e_{t+2} + e_{t+1}}{2}\right), \left(\frac{e_{t+1} + 2e_t + e_{t-1}}{2}\right)\right].$$

Since the e's are uncorrelated with each other and since e_{t+1} is the only value of e that appears in both expressions, the covariance reduces to

(8) $$\text{cov}\left[\left(\frac{C_{t+2} + C_{t+3}}{2} - \frac{C_t + C_{t+1}}{2}\right), \left(\frac{C_t + C_{t+1}}{2} - \frac{C_{t-2} + C_{t-1}}{2}\right)\right] = \frac{\sigma_e^2}{4},$$

where σ_e^2 denotes the variance of the e's. So the change in measured consumption is correlated with its previous value. Since the covariance is positive, this means that if measured consumption in the two-period interval $(t, t + 1)$ is greater than measured consumption in the two-period interval $(t - 2, t - 1)$, measured consumption in $(t + 2, t + 3)$ will tend to be greater than measured consumption in $(t, t + 1)$. When a variable follows a random walk, successive changes in the variable are uncorrelated. For example, with actual consumption in this model, we have $C_t - C_{t-1} = e_t$ and $C_{t+1} - C_t = e_{t+1}$. Since e_t and e_{t+1} are uncorrelated, successive changes in actual consumption are uncorrelated. Thus if C_t were bigger than C_{t-1}, it would not mean that C_{t+1} would tend to be higher than C_t. Since successive changes in measured consumption are correlated, measured consumption is not a random walk. The change in measured consumption today does provide us with some information as to what the change in measured consumption is likely to be tomorrow.

(c) From equation (5), the change in measured consumption from $(t, t + 1)$ to $(t + 2, t + 3)$ depends on e_{t+1}, the innovation to consumption in period $t + 1$. But this is known as of $t + 1$, which is part of the first two-period interval. Thus the change in consumption from one two-period interval to the next is not uncorrelated with everything known as of the first two-period interval. However, it is uncorrelated with everything known in the two-period interval immediately preceding $(t, t + 1)$. From equation (5), e_{t+3}, e_{t+2} and e_{t+1} are all unknown as of the two-period interval $(t - 2, t - 1)$.

II – Ricardian Equivalence and Income Unvertainty

Consider a two-period economy. Individuals receive income ω_1 in period 1 and ω_2 in period 2. ω_1 is known and $\omega_2 = \omega_1 + \varepsilon$ is random. ε is a random variable with zero mean and whcih is uncorrelated accross individuals, so that there is no aggregate uncertainty. We therefore have $E[\omega_2] = \omega_1$. We assume individuals can borrow or lend at rate $R = 1$. Their utility is $u(c_1) + E[u(c_2)]$, with $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u'''(\cdot) > 0$.

In some questions, we will assume that utility is quadratic, so that $u(c) = c - bc^2$ with $b > 0$. We will assume then that $c < \frac{1}{2b}$, so that $u' > 0$.

The government taxes income at rate t_1 and t_2 in the two periods.

1 – Find the Euler equation of an individual that maximises expected utility under an intertemporal budget constraint (to be written).

Intertemporal budget constraint is

$$c_2 = (1 - t_1)\omega_1 + (1 - t_2)(\omega_2 + \varepsilon) - c_1$$

and Euler equation is

$$u'(c_1) = E[u'(c_2)]$$

2 – Show that $E[c_2] = c_1$ when utility is quadratic.

$u'(c) = 1 - 2bc$ and the Euler equation implies $c_1 = c_2$.

3 – Show that $E[c_2] > c_1$ when $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u'''(\cdot) > 0$. Why do we refer to that case as a case with *precautionary saving*?

As $u'''(c) < 0$, we have by Jensen inequality $E[u'(c_2)] > u'(E[c_2])$. The Euler equation is

$$u'(c_1) = E[u'(c_2)].$$

Therefore

$$E[u'(c_2)] > u'(E[c_2])$$

so that

$$E[c_2] > c_1.$$

4 – Consider now a marginal tax change $dt_1 < 0$ and $dt_2 > 0$ that leaves expected tax revenues constant. What is the value of $\frac{dt_2}{dt_1}$?

By differentiating the Euler equation with respect to c_1 and t_1 (using the intertemporal budget constraint to eliminate c_2 and the fact that expected revenues are constant), find an expression for $\frac{dc_1}{dt_1}$, which is the marginal propensity to consume out of a tax cut. You will have to use the equality $E[xy] = E[x]E[y] + \text{cov}(x, y)$ for two random variables x and y.

Expected tax revenues are

$$R = t_1\omega_1 + t_2 E[\omega_2] = (t_1 + t_2)\omega_1$$

and $dR = 0 \Leftrightarrow dt_2 = -dt_1$.
Now take the Euler equation

$$u'(c_1) = E\Big[u'((1 - t_1)\omega_1 - c_1 + (1 - t_2)(\omega_1 + \varepsilon))\Big]$$

Differentiate to obtain

$$u''(c_1)dC_1 = E\Big[u''(c_2)\Big(-\omega_1 dt_1 - dc_1 - (\omega_1 + \varepsilon)\frac{dt_2}{dt_1}dt_1\Big)\Big]$$

so that

$$(u''(c_1) + E[u''(c_2)])\frac{dc_1}{dt_1} = E[u''(c_2)\varepsilon].$$

Therefore,

$$(u''(c_1) + E[u''(c_2)])\frac{dc_1}{dt_1} = E[u''(c_2)]\underbrace{E[\varepsilon]}_{=0} + \text{cov}(u''(c_2), \varepsilon)$$

so that

$$\frac{dc_1}{dt_1} = \frac{\text{cov}(u''(c_2), \varepsilon)}{u''(c_1) + E[u''(c_2)]}$$

5 – Show that $\frac{dc_1}{dt_1} = 0$ when utility is quadratic. What does it imply for Ricardian equivalence?

If u is quadratic, u'' is a constant, so that $\text{cov}(u''(c_2), \varepsilon) = 0$, and therefore $\frac{dc_1}{dt_1} = 0$.

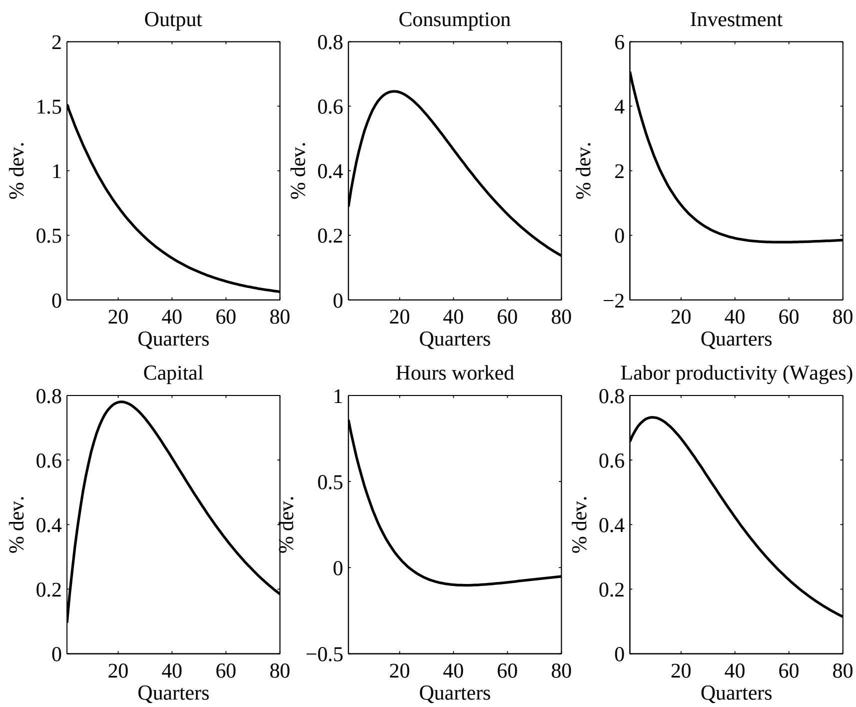
6 – Explain why $\text{cov}(c_2, \varepsilon)$ is positive. Use that result to show that $\frac{dc_1}{dt_1} < 0$ when when $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u'''(\cdot) > 0$. Interpret the result.

As c_1 is decided before the realization of ε, the intertemporal budget cinstraint implies that c_2 will increase with ε. As $u''' > 0$, $\text{cov}(c_2, \varepsilon)$. As the denominator $u''(c_1) + E[u''(c_2)]$ is negative, we have $\frac{dc_1}{dt_1} < 0$.

III – The Mechanisms of the "RBC" Model

Figure 1 below displays the responses of some macroeconomic variables to a persistent but not permanent Total Factor Productivity shock in a version of the Lecture 9 model with capital and elastic labor supply. Explain how those responses are explained by consumption smoothing, consumption/leisure substitution and general equilibrium.

Figure 1: Response to a persistent TFP shock



(a) There are two main economic mechanisms that are driving this dynamics: *(i)* Consumption smoothing : preferences are convex so that agents prefer to smooth consumption over time. Therefore, an increase in today supply of good will not be all consumed today, but partly saved. *(ii)* Consumption/leisure substitution: consumption and leisure are two normal goods, whose demand increases when income increase or price decrease. The relative price of leisure is the real wage. Therefore, changes in the real wake will affect labor supply through an income and substitution effect.
(b) Given those economic motives, there are two sources of dynamics in the model: *(i)* The dynamics of the shock: TFP is assumed to be persistent. A shock today as long lasting effects, although non permanent. *(ii)* The law of motion of capital: capital can be carried from one period to another, and is the way to transform current goods into future goods.
This is well seen in the case of the analytic RBC model studied in the problem set, whose solution is

$$\begin{aligned} y_{t+1} &= \alpha y_t + z_{t+1} + \alpha \log \alpha\beta \\ z_{t+1} &= z_t + \varepsilon_{t+1} \end{aligned}$$

where z is the exogenous TFP. Even if z was *iid*, there would be some persistence on the shocks.
(c) The story of those IRF is therefore: TFP increases, but not permanently. This creates a wealth effect as existing capital and labor are more productive *ceteris paribus.* As consumption and output are normal goods, we should expect both to increase. This does not happen to leisure because at the same time leisure becomes more expensive (because marginal productivity of labor increases). This second effect dominates and labor goes up. Not all this extra production (y is increasing) is going into consumption, and part is saved. hence, i increases and so does k. Then all converges back to the steady state.
(d) Finally, note how prices can decentralize those allocations: the real wage is going up (with Cobb-Douglas technology and perfect competition, the real wage moves proportionally to average productivity of labor). The real interest rate (not shown on this graph) is also going up in the short run, so that agents save part of their extra income.