

**UNIVERSITY COLLEGE LONDON**

**EXAMINATION FOR INTERNAL STUDENTS**

MODULE CODE : **ECON0107**  
ASSESSMENT : **Mid Term – 24 Hour Online**  
PATTERN  
MODULE NAME : **Macroeconomics**  
LEVEL: : **Postgraduate**  
DATE : **Wed 12<sup>th</sup> Jan 2022**  
TIME : **12:00**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

**Year  
2021/22**

<b>Additional material</b>	
<b>Special instructions</b>	
<b>Exam paper word count</b>	

**TURN OVER**

MIDTERM COURSEWORK ASSESSMENT

*Problem I carries 50% of the total mark, Problem II 40% and Problem III 10%.  
Students are expected to spend 4 hours on the paper.*

By submitting this assessment, I pledge my honour that I have not violated UCL's Assessment Regulations which are detailed in the [UCL academic manual](#) (chapter 6, section 9 on student academic misconduct procedure), which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.

I – A MODEL WITH HETEROGENOUS AGENTS

Assume there is a continuum of households, each have quadratic felicity in consumption  $u(c) = \alpha_1 c - \alpha_2 c^2/2$ . Households maximize the discounted stream of felicity, discounted by factor  $\beta$ . Each household faces stochastic income  $y$ , but the aggregate income is constant. There is no liquidity constraint and households can save and dissave in a riskless bond  $a$  at return  $R$  (with a No-Ponzi condition). The budget constraint reads:  $a_{t+1} = R(a_t + y_t - c_t)$ .

**A. Household Behavior**

- 1 – Set up the household problem and derive the Euler equation. Argue that consumption is a martingale, if the interest rate always equals the time-preference rate.
- 2 – Derive the intertemporal or net present value budget constraint and use it to derive the consumption function (assume that the interest rate always equals the time- preference rate for questions 2, 3 and 4).
- 3 – Show that the change in consumption,  $c_{t+1} - c_t$ , is related to the news the household receives about future income. Show that consumption is a martingale using this condition.
- 4 – Now assume that income follows an autoregressive process:  $y_{t+1} = \rho y_t + \eta_{t+1}$ ;  $0 < \rho \leq 1$ . Derive an expression for the change in consumption and how it depends on  $\eta_{t+1}$ . Is consumption a martingale? Is it feasible to test this in data? What is the disadvantage relative to Hall (1978)'s approach.

**B. Equilibrium**

- 5 – Argue that the time preference rate is the equilibrium interest rate in this economy, when the bond market needs to clear at aggregate holdings  $\int a = 0$ .
- 6 – Suppose all income shocks are i.i.d. Is the equilibrium in question 5 a stationary one?
- 7 – Now assume that there is an aggregate endowment increase in one specific period (known in advance). Will that affect aggregate consumption in other periods in equilibrium? How can it affect individual consumption (it suffices to extrapolate, no maths!)?

**C. Measurement**

The above model implies:  $c_t = c_{t-1} + e_t$ , where  $e$  is white noise. Assume that we only observe average consumption over two-period intervals in the data; that is,  $(c_t + c_{t+1})/2$ ,  $(c_{t+2} + c_{t+3})/2$ , and so on.

- 8** – Find an expression for the change in measured consumption from one two-period interval to the next in terms of the  $e$ 's.
- 9** – Is the change in measured consumption uncorrelated with the previous value of the change in measured consumption? In light of this, is measured consumption a random walk?
- 10** – Given your result in question 1, is the change in consumption from one two- period interval to the next necessarily uncorrelated with anything known as of the first of these two-period intervals? Is it necessarily uncorrelated with anything known as of the two-period interval immediately preceding the first of the two-period intervals?

## II – RICARDIAN EQUIVALENCE AND INCOME UNVERTAINTY

Consider a two-period economy. Individuals receive income  $\omega_1$  in period 1 and  $\omega_2$  in period 2.  $\omega_1$  is known and  $\omega_2 = \omega_1 + \varepsilon$  is random.  $\varepsilon$  is a random variable with zero mean and which is uncorrelated across individuals, so that there is no aggregate uncertainty. We therefore have  $E[\omega_2] = \omega_1$ . We assume individuals can borrow or lend at rate  $R = 1$ . Their utility is  $u(c_1) + E[u(c_2)]$ , with  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$  and  $u'''(\cdot) > 0$ .

In some questions, we will assume that utility is quadratic, so that  $u(c) = c - bc^2$  with  $b > 0$ . We will assume then that  $c < \frac{1}{2b}$ , so that  $u' > 0$ .

The government taxes income at rate  $t_1$  and  $t_2$  in the two periods.

- 1** – Find the Euler equation of an individual that maximises expected utility under an intertemporal budget constraint (to be written).
- 2** – Show that  $E[c_2] = c_1$  when utility is quadratic.
- 3** – Show that  $E[c_2] > c_1$  when  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$  and  $u'''(\cdot) > 0$ . Why do we refer to that case as a case with *precautionary saving*?
- 4** – Consider now a marginal tax change  $dt_1 < 0$  and  $dt_2 > 0$  that leaves expected tax revenues constant. What is the value of  $\frac{dt_2}{dt_1}$ ?
- By differentiating the Euler equation with respect to  $c_1$  and  $t_1$  (using the intertemporal budget constraint to eliminate  $c_2$  and the fact that expected revenues are constant), find an expression for  $\frac{dc_1}{dt_1}$ , which is the marginal propensity to consume out of a tax cut. You will have to use the equality  $E[xy] = E[x]E[y] + \text{cov}(x, y)$  for two random variables  $x$  and  $y$ .
- 5** – Show that  $\frac{dc_1}{dt_1} = 0$  when utility is quadratic. What does it imply for Ricardian equivalence?
- 6** – Explain why  $\text{cov}(c_2, \varepsilon)$  is positive. Use that result to show that  $\frac{dc_1}{dt_1} < 0$  when  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$  and  $u'''(\cdot) > 0$ . Interpret the result.

## III – THE MECHANISMS OF THE “RBC” MODEL

Figure 1 below displays the responses of some macroeconomic variables to a persistent but not permanent Total Factor Productivity shock in a version of the Lecture 9 model with capital and elastic labor supply. Explain how those responses are explained by consumption smoothing, consumption/leisure substitution and general equilibrium.

Figure 1: Response to a persistent TFP shock

