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The Welfare Cost of Business Cycles in an Economy with Nonclearing Markets

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Abstract

In this paper we measure the welfare cost of fluctuations in a simple representative agent economy with nonclearing markets. The market friction we consider involves price rigidities and a voluntary exchange rationing scheme. These features are incorporated into an otherwise standard neoclassical growth model. We show that the frictions we introduce make the losses from fluctuations four times bigger than in a frictionless environment.

KEYWORDS: cost of business cycles, nonclearing markets, dynamic general equilibrium

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1 Introduction

In a seminal contribution, Lucas [1987] has shown that in a representative agent framework, the welfare gain from stabilizing consumption around its mean is small. Let us recall briefly Lucas's argument. Assume that aggregate consumption follows a log linear process around a deterministic trend, $c_t = (1+\mu)^t e^{-\frac{1}{2}\sigma_z^2} z_t$, where $\{z_t\}$ is a stationary stochastic process with a stationary distribution given by $\ln z_t \rightsquigarrow N(0, \sigma_z^2)$, so that the expected value of consumption does not depend on the variance. Then, the cost of instability can be computed as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption instability and a perfectly smooth consumption path. With a CRRA utility function with risk aversion coefficient ν, this cost is given by $\frac{1}{2}\nu\sigma_z^2$. With $\sigma_z = 0.013$ (Lucas's estimate), and $\nu = 5$, the welfare cost of fluctuations is only 0.042% of average consumption. When implemented in calibrated versions of standard representative-agent Dynamic Stochastic General Equilibrium (DSGE) models, the conclusion is basically unchanged.[1]

In this paper we propose a model in which the welfare cost of fluctuations is non trivial, because fluctuations magnify some market imperfections. The market friction we consider is the predetermination of some prices. When prices are set in advance, markets do not clear, and we assume that transactions occur at the minimum of demand and supply. Such a rationing scheme is known as "voluntary exchange hypothesis" in the literature on nonclearing markets (see Benassy [1993] for an overview), and it is a very natural one in a free market economy: no agent can be forced to purchase more than she demands, or to sell more than she supplies. As in Lucas [1987] and Lucas [2003], our approach consists of measuring the costs to risk averse households of the consumption variability associated with the business cycle.[2] As we know the model economy we adopt a structural measure of the welfare cost of fluctuations. We discuss this measure and its relation with a measure that

[1] Two strands of the literature have looked for DSGE models in which Lucas's measure can be higher. The first relaxes the assumption of a representative agent and introduces incomplete insurance markets as in Imrohoroglu [1988], Atkeson and Phelan [1994] and Krusell and Smith [1999] among others. A second strand, following the work of Epstein and Zin [1991], adopts more general utility functions, for which the intertemporal elasticity of substitution is not the inverse of the degree of relative risk aversion, as in Obstfeld [1994] and Epaulard and Pommeret [2003] among others.

[2] As it is standard in this context we are restricting ourselves to the question about the welfare gains of eliminating business cycles which is truly a hypothetical one. The limitation of doing so is that the exercise is silent about the design of policy that would stabilize the economy.

only requires the knowledge of the equilibrium process of consumption in some detail.

We show that nonclearing markets make the losses from fluctuations much bigger than in a frictionless environment. But this is so only when the welfare cost of fluctuations is measured taking fully account of transitions and nonlinearities. In order to make the argument more transparent we restrict ourselves to a fully analytically computable case.

The paper is organized as follows. In Section 2 we present a simple analytical Real Business Cycle model with or without nonclearing markets. Section 3 describes the way we measure welfare costs of fluctuations. In Section 4 we present our main quantitative results. A last section concludes.

2 The Model Economy

We first introduce the environment, and then describe the two cases we consider: the walrasian case and the nonclearing market case.

The economy is competitive, populated with a large number of identical households and firms. All agents behave competitively and have rational expectations. The final good is the numeraire. The household buys the consumption good, accumulates capital, produces an intermediate good using its own labor, sells intermediate good at price ω_t and rents capital at price z_t to the final good firm. The final good firm sells its output to the household, that allocates it between consumption and investment (next period capital). The production function of the final good firm is subject to technology shocks. The intermediate and final good cannot be stored.

The non-walrasian feature of the model comes from the fact that the price of the intermediate good is set before observation of the technology shock. As we want to maintain the perfect competition assumption, we follow the fixed price literature (see for example Benassy [1995]) and assume that the price of the intermediate good is set at the level that clears the intermediate good market in expectation. It should be clear that this is an *ad hoc* assumption. It has the advantage that, absent of shocks, one recovers the walrasian allocation. The production of the intermediate good is done before observation of the technology shock. Therefore, in case of a negative surprise on the level of technology, a fraction of the intermediate good will not be sold, and will be wasted, although the amount wasted is zero in expectations. Once the shock is observed, the final good firm determines its optimal demand for intermediate good, and trade occurs. As prices are not walrasian, supply and demand will generically not equalize, and a rationing scheme must be specified. We assume voluntary exchange: no agent is forced to buy or sell if she does not want to. As a consequence, the level of transaction on the intermediate good market will be the minimum of demand and supply at given (non walrasian) price. If demand is lower than supply, the household's marginal proceeds of the intermediate good sale is *ex post* lower than the marginal disutility of labor, although it was *ex ante* equal. Without shocks, the preset price economy replicates the competitive allocations, and is therefore efficient. Fluctuations are costly not only because agents prefer smooth paths for consumption and leisure, as in the competitive model, but also because fluctuations create inefficiencies and a waste of resources.

The calibrated model we consider is not the most realistic, as we restrict ourselves to a fully analytically computable case. Indeed, we will consider a case with logarithmic utility, lognormal shocks and full depreciation.

2.1 Technology, Preferences and Markets

Lower-case letters denote individual quantities and upper-case letters aggregate ones. Utility is derived from consumption and leisure, the intermediate good is produced with labor, while the final good is produced using capital and the intermediate good. The final good is either consumed or invested.

More specifically, the final good firm uses x_t units of the intermediate good and k_t units of capital to produce according to a Cobb Douglas technology:

$$y_t = \theta_t k_t^{\alpha} x_t^{1-\alpha} \tag{1}$$

with

$$\theta_t = \Theta(\sigma_\varepsilon, \rho)\theta_{t-1}^{\rho}\varepsilon_t. \tag{2}$$

Here, ε_t is the innovation to θ_t, and it is assumed that $\log \varepsilon_t$ follows an *i.i.d.* Gaussian process with zero mean and standard deviation σ_ϵ. We also assume that $|\rho| < 1$. $\Theta(\sigma_\varepsilon, \rho) = \left(\exp\left(-\frac{1}{2}\frac{\sigma_\varepsilon^2}{1-\rho^2}\right)\right)^{1-\rho}$ is a correction parameter that guarantees that the mean of θ is always equal to one, for any level of σ_ε and ρ. Therefore, variations in the level of σ_ε will be mean preserving spreads to the distribution of θ. It is useful for the following to denote by F the c.d.f. of θ, and to note that $\theta_t = E_{t-1}[\theta_t] \times \varepsilon_t$.

The final good cannot be stored, is used for consumption and savings by the mean of capital accumulation. Capital fully depreciates from one period to another. Aggregate capital is predetermined, and equal to the last period savings. The resource constraint of the economy is

$$C_t + K_{t+1} \leq Y_t. \tag{3}$$

The household transforms its own labor n_t into intermediate good x_t according to the linear one-to-one technology

$$x_t = n_t. \tag{4}$$

The intermediate good cannot be stored.

Preferences are given by the following expected discounted lifetime utility

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\log c_{t+j} + \gamma \log(1 - n_{t+j}) \right) \right]. \tag{5}$$

E_t is the conditional mathematical expectation operator, where the information set includes all current and past variables of the economy. It is assumed that all agents behave in a competitive way. The household consumes c_t, saves a_{t+1}, produces intermediate good x_t^s with labor n_t, sells the intermediate good and rents its accumulated capital a_t. The final good firm uses the intermediate good x_t and capital k_t to produce, and sells goods y_t.

We consider two cases. In the first one (the Walrasian case), there are no preset prices. In the second case (the nonclearing markets case), the price ω_t of the intermediate good is preset and the production of the intermediate good is realized before the current shock has been observed. Therefore, it would be useful for the intermediate good (actually, for any traded good) to distinguish between individual demand and supply, x^d and x^s. Correspondingly between aggregate quantities, X^d and X^s, and transactions X.

2.2 The Walrasian Case

In this case, all decisions are taken after the shock ε_t has been revealed.

Optimal Individual Behavior : The final good firm maximizes her profit $y_t - \omega_t x_t^d - z_t k_t$ for given walrasian input prices ω_t (intermediate good) and z_t (capital). From this problem, one gets the two following first order conditions:

$$z_t = \alpha \theta_t k_t^{\alpha-1} (x_t^d)^{1-\alpha} \tag{6}$$

$$\omega_t = (1-\alpha) \theta_t k_t^{\alpha} (x_t^d)^{-\alpha} \tag{7}$$

from which we derive capital demand k and intermediate good demand x^d.

The household maximizes her utility with respect to an intertemporal budget constraint. This problem admits the following recursive representation:

$$V(a, \theta, K) = \max_{c,n,a',x^s} \{(\log c + \gamma \log(1-n)) + \beta E V(a', \theta', K')\} \tag{8}$$

$$\text{s.t.} \quad \left\{ \begin{array}{lcl} c + a' & \leq & za + \omega x^s \\ x^s & \leq & n. \end{array} \right.$$

In any period t, the first order conditions of this problem are given by

$$\begin{aligned} \frac{\gamma}{1-n_t} &= \frac{\omega_t}{c_t} & (9) \\ \frac{1}{c_t} &= \beta E_t\left[\frac{z_{t+1}}{c_{t+1}}\right] & (10) \\ x_t^s &= n_t. & (11) \end{aligned}$$

Walrasian Equilibrium : By definition, a *Walrasian Equilibrium* consists of a household value function $V(a,\theta,K)$, a household policy $\{c(a,\theta,K), n(a,\theta,K), x^s(a,\theta,K),\ a'(a,\theta,K)\}$, a firm policy $\left\{y(\theta,K), k(\theta,K), x^d(\theta,K)\right\}$, prices ω and z, and aggregate quantities C, K, A, Y, N and X such that *(i)* at given prices, household value function and policy solve the decision problem (8), *(ii)* firm policy solves the firm profit maximization problem, *(iii)* aggregate quantities are equal to their individual counterparts (under the representative agent assumption), and *(iv)* prices are such that markets clear.

Walrasian allocations can be analytically computed as follows: from (10) and (6), and using (3), we obtain

$$\frac{K_{t+1}}{C_t} = \alpha\beta E_t\left[1+\frac{K_{t+2}}{C_{t+1}}\right]. \tag{12}$$

Solving (12) forward, using (3) and the transversality condition, we obtain

$$\begin{aligned} C_t &= (1-\alpha\beta)Y_t & (13) \\ K_{t+1} &= \alpha\beta Y_t. & (14) \end{aligned}$$

In this model, the saving rate is constant, which is key to get an analytical solution. Combining (9) with (7) and (13) gives

$$X_t = \widetilde{X} = N_t = \widetilde{N} = \frac{(1-\alpha)}{1-\alpha+\gamma(1-\alpha\beta)}. \tag{15}$$

According to (15), employment is constant in a competitive equilibrium.

In the walrasian case, the equilibrium dynamics is therefore fully characterized by the following set of equations:

$$\left\{\begin{array}{rcl} \theta_t & = & \Theta(\sigma_\varepsilon,\rho)\theta_{t-1}^{\rho}\varepsilon_t \\ N_t & = & \widetilde{N} \\ X_t & = & \widetilde{X} \\ Y_t & = & \theta_t K_t^{\alpha} X_t^{1-\alpha} \\ C_t & = & (1-\alpha\beta)Y_t \\ A_t & = & K_t \\ K_{t+1} & = & \alpha\beta Y_t \\ \omega_t & = & (1-\alpha)\theta_t K_t^{\alpha} X_t^{-\alpha} \\ z_t & = & \alpha\theta_t K_t^{\alpha-1} X_t^{1-\alpha}. \end{array}\right. \tag{16}$$

2.3 The Nonclearing Market Case

Each period is divided into two subperiods. In the first one, the shock ε_t is unknown, production of the intermediate good takes place and the price ω is set. In the second subperiod, ε_t realizes and transactions take place. In this non-walrasian setting, demand does not necessary equal supply, and transactions are set at the minimum of supply and demand when a market does not clear. Uniform rationing across firms is assumed.

Second Subperiod Choices : In the second subperiod, the amount of intermediate goods produced is already set and the shock ε_t is revealed. The representative final good firm behaves in a competitive way, but might be rationed on the intermediate good market. Her program is

$$\max_{k_t, x_t^d} \quad y_t - z_t k_t - \omega_t x_t^d$$

$$\text{s.t.} \quad \left\{\begin{array}{rcll} y_t & \leq & \theta_t k_t^{\alpha}(x_t^d)^{1-\alpha} & \\ x_t^d & \leq & \overline{x}_t^s & (\nu_t) \end{array}\right.$$

where $\overline{x}_t^s$ is a quantity constraint that the firm faces on the intermediate good market, and ν_t is the Lagrange multiplier associated to this constraint.

The household enters that subperiod with a given stock of intermediate good (equal to n_t), and supplies as much as can be sold of it at preset price ω_t. Once received the proceeds of the intermediate good sales and the rental rate on its capital, she decides to allocate income between consumption and investment. She solves a problem whose value function $\widehat{V}_2$ is:

$$\widehat{V}_2(a,n,\omega,\theta,K) = \max_{c,x^s,a'} \left\{ \log c + \beta E \widehat{V}_1(a',\theta',K') \right\} \tag{17}$$

$$\text{s.t.} \quad c + a' \leq za + \omega x^s \tag{18}$$

$$x^s \leq \overline{x}^d \tag{19}$$

$$x^s \leq n \tag{20}$$

where $\overline{x}^d$ is a quantity constraint that the household might face. $\widehat{V}_1$ is the beginning of first subperiod value, that we formally define next.

First Subperiod Household Problem : In that subperiod, the price ω_t at which the intermediate good will be sold to the final good firm is set, and the household decides how much to work in the production of the intermediate good, taking into account that it might be constrained on its intermediate good sales in the second subperiod. The final good firm does not take any decision in that subperiod. The household's intermediate good production optimal behavior is derived from the problem:

$$\widehat{V}_1(a,\theta,K) = \max_n \left\{ \gamma \log(1-n) + \int_\theta \widehat{V}_2(a,n,\omega,\theta,K) \mathrm{d}F(\theta) \right\}. \tag{21}$$

First Order Conditions : Solving the firm maximization problem, capital services and intermediate good demands are obtained from the following first order conditions

$$\begin{cases} z_t = \alpha \theta_t k_t^{\alpha-1} (x_t^d)^{1-\alpha} \\ \omega_t = (1-\alpha)\theta_t k_t^\alpha (x_t^d)^{-\alpha} - \nu_t. \end{cases}$$

There are two regimes, depending on whether the quantity constraint is binding or not ($\nu_t > 0$ or $\nu_t = 0$). It is useful for the following to compute optimal intermediate good demand when the quantity constraint is not binding. This demand, denoted $x_t^{d\star}$, is given by:

$$x_t^{d\star} = \omega_t^{-1/\alpha}(1-\alpha)^{1/\alpha}\theta_t^{1/\alpha}k_t. \tag{22}$$

The household first order conditions are now derived. For the first subperiod, the optimality condition of the problem (21) is

$$\frac{\gamma}{1-n_t} = \int_\theta \frac{\partial \widehat{V}_2}{\partial n_t} \mathrm{d}F(\theta). \tag{23}$$

In order to deal with the quantity constraint (19), it is convenient to define $\widehat{V}_{2,nc}$ (for "not constrained") and $\widehat{V}_{2,c}$ (for "constrained") as the second subperiod value functions when the constraint (19) is binding or not binding. Note that when (19) does not bind, (20) does.

$$\begin{aligned} \widehat{V}_{2,nc}(a,n,\omega,\theta,K) &= \max_{c,a'} \left\{ \log c + \beta E \widehat{V}_1(a',\theta',K') \right\} \\ & \text{s.t.} \quad c + a' \leq za + \omega n \end{aligned} \tag{24}$$

and

$$\begin{aligned} \widehat{V}_{2,c}(a,n,\omega,\theta,K) &= \max_{c,a'} \left\{ \log c + \beta E \widehat{V}_1(a',\theta',K') \right\} \\ & \text{s.t.} \quad c + a' \leq za + \omega \overline{x}^d. \end{aligned} \tag{25}$$

From (21) and the definitions of the value functions in (24) and (25), it is useful to note that

$$\frac{\partial \widehat{V}_{2,nc}}{\partial n} = \frac{\omega}{c} \tag{26}$$

$$\frac{\partial \widehat{V}_{2,c}}{\partial n} = 0. \tag{27}$$

Equation (27) shows that the marginal gain from supplying labor and producing intermediate good is zero when the household is constrained on the intermediate good market.

Finally, we derive from the problem (17) the first order condition of the household consumption/saving problem in the second subperiod, which is given by

$$\frac{1}{c_t} = \beta E_t \left[\frac{z_{t+1}}{c_{t+1}} \right]. \tag{28}$$

Preset Price : We have assumed that ω_t is preset at the level that clears the intermediate good market at period t in expectation. For a given level of capital K_t, this price is the expected walrasian price, as given in (16), so that

$$\omega_t = \int_\theta (1-\alpha) K_t^\alpha \widetilde{N}^{-\alpha} \theta_t \mathrm{d}F(\theta) = (1-\alpha) K_t^\alpha \widetilde{N}^{-\alpha} E_{t-1}[\theta_t], \tag{29}$$

where $\widetilde{N}$ is the constant equilibrium level of employment in the walrasian equilibrium.

Nonclearing Market Equilibrium : By definition, a *Nonclearing Market Equilibrium* consists of household value functions $\widehat{V}_1(a,\theta,K)$ and $\widehat{V}_2(a,n,\omega,\theta,K)$, a household policy $\{c(a,\theta,K), n(a,\theta,K), x^s(a,\theta,K), a'(a,\theta,K)\}$, a firm policy $\{y(\theta,K), k(\theta,K), x^d(\theta,K)\}$, quantity constraints $\overline{x}^d$ and $\overline{x}^s$, prices ω and z, and aggregate quantities C, K, A, Y, N, $\overline{X}^d$, $\overline{X}^s$ and X such that *(i)* at given prices, household value functions and policy solve the decision problems (21) and (17), *(ii)* firm policy solves the firm profit maximization problem, *(iii)* aggregate quantities are equal to their individual counterparts (under the representative agent assumption), *(iv)* $\overline{X}^d = X^d$, $\overline{X}^s = X^s$, ω satisfies (29) and transactions X are at the minimum of demand and supply on the intermediate good market, and *(v)* the good and capital services markets clear.

Nonclearing market allocations can be analytically computed as follows. We first derive the non walrasian equilibrium employment, and then compute other equilibrium quantities. From the expression of the non constrained intermediate input demand (22) and the value of the ω_t, as given in (29), we get

$$X_t^{d^\star} = \widetilde{N}\varepsilon_t^{1/\alpha}. \tag{30}$$

From the voluntary exchange assumption, we obtain that transactions on the intermediate good market are given by

$$X_t = \min\left(N_t, \widetilde{N}\varepsilon_t^{1/\alpha}\right). \tag{31}$$

Therefore, the household will be constrained on the intermediate good market when $\widetilde{N}\varepsilon_t^{1/\alpha} \leq N_t$, that is when ε_t belongs to the set $\mathcal{E}(N_t) = \{\varepsilon_t$ such that $\log\varepsilon_t \leq \alpha(\log N_t - \log\widetilde{N})\}$. Now using the labor supply equation (23), we obtain

$$\frac{\gamma}{1-N_t} = \int_{\varepsilon_t \notin \mathcal{E}(N_t)} \frac{(1-\alpha)\widetilde{N}^{-\alpha}E_{t-1}\theta_t}{(1-\alpha\beta)N_t^{1-\alpha}\theta_t}\mathrm{d}G(\varepsilon_t) \tag{32}$$

where G is the cumulative of the Normal distribution. Manipulating this expression and making use of the definition of $\mathcal{E}(N_t)$, we finally obtain

$$\frac{\gamma N_t^{1-\alpha}}{1-N_t} = \frac{1-\alpha}{1-\alpha\beta}\widetilde{N}^{-\alpha}\int_{\left(\frac{N_t}{N}\right)^\alpha}^{+\infty}\frac{1}{\varepsilon_t}\mathrm{d}G(\varepsilon_t). \tag{33}$$

Equation (33) deserves some comments. First, the solution does not depend on time, as ε is an *iid* shock. Therefore, we will have $N_t = \overline{N}\ \forall\ t$ in the nonclearing market equilibrium. Second, without shocks, *i.e.* if $\varepsilon_t = 1\ \ \forall\, t$, the preset price ω_t is the walrasian one, and one can check that $\overline{N} = \widetilde{N}$. Third, (33) implies that $\overline{N} < \widetilde{N}$: as there exists some uncertainty on the second subperiod productivity when the household supplies hours, a precautionary motive leads the household to lower labor supply and therefore employment. Equation (33) has no analytical solution, but its solution can be easily computed numerically.

Finally, using (28), the budget constraint and the transversality condition, we obtain that the saving rate is also constant in the nonclearing market economy, and therefore (13) and (14) hold. The rest of the model is then solved trivially.

In the nonclearing markets case, the dynamics of aggregate quantities is therefore fully characterized by the following set of equations:

$$\left\{\begin{array}{rcl} \theta_t & = & \Theta(\sigma_\varepsilon,\rho)\theta_{t-1}^{\rho}\varepsilon_t \\ N_t & = & \overline{N} \\ X_t & = & \min\left(\overline{N}, \widetilde{N}\varepsilon_t^{1/\alpha}\right) \\ Y_t & = & \theta_t K_t^{\alpha} X_t^{1-\alpha} \\ C_t & = & (1-\alpha\beta)Y_t \\ A_t & = & K_t \\ K_{t+1} & = & \alpha\beta Y_t \\ \omega_t & = & (1-\alpha)K_{t-1}^{\alpha}\widetilde{N}^{-\alpha}E_{t-1}[\theta_t] \\ z_t & = & \alpha\theta_t K_t^{\alpha-1}X_t^{1-\alpha}. \end{array}\right. \tag{34}$$

3 Welfare Analysis

3.1 Measuring Costs

To obtain a structural (model driven) measure of the welfare cost of fluctuations, we compare the economies with and without fluctuations starting from the same set of initial conditions $S = (K, \theta)$. The measure we compute can be understood as the outcome of the following thought experiment of structural change: let us assume that we have been in an economy with shocks from $-\infty$ to $T-1$, and that from T to ∞, fluctuations are eliminated by setting $\varepsilon_t = 1 \ \ \forall\, t \geq T$. We evaluate the welfare gain of this structural change by comparing the expected intertemporal utility of the representative agent in two economies: an economy A that starts with initial condition S_{T-1} and in which shocks are not shut down; an economy B that starts with initial condition S_{T-1} and without shocks. The conditional (on S_{T-1}) welfare cost of fluctuations $\mathcal{C}(S_{T-1})$ is then defined as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption path A and B. By repeating this experiment for many different starting points, drawn in the ergodic distribution of the economy with shocks, one will get an unconditional measure of the welfare cost of fluctuations $\mathcal{C} = E\left[\mathcal{C}(S_{T-1})\right]$. More formally, the measure we propose is given by

$$\int_{S_{T-1}} \int_{\mathcal{E}} \sum_{j=0}^{\infty} \beta^j \left[\log\left(C^A_{T+j}\left(1+\mathcal{C}\times C_{SS}\right)\right) + \gamma \log(1-N^A_{T+j})\right] \, \mathrm{d}g(\mathbf{e}) \, \mathrm{d}f(S)$$

or equivalently

$$\int_{S_{T-1}} \sum_{j=0}^{\infty} \beta^j \left[\log C^B_{T+j} + \gamma \log(1-N^B_{T+j})\right] \, \mathrm{d}f(S). \tag{35}$$

where $\mathbf{e}$ is an infinite sequence of ε and f the ergodic joint density of (K,θ) in the economy with shocks. Note that $\mathcal{C}$ is expressed in percentage points of the non stochastic steady-state level of consumption C_{SS}. Next we explain the way in which we implement the unconditional measure of the welfare cost of fluctuations that we refer to as a comprehensive one.

3.2 Computation of the Welfare Cost of Fluctuations

The first step consists of computing the solution of the model. This is immediate in our simple analytical case but it should be obtained by accurate computational methods in most DSGE models. Note that even though we are able to obtain an analytical solution to the model in the nonclearing market case, we cannot compute its moments analytically, because of its nonlinearity (the min operator). We then simulate the solution of the model over 45.000 periods and build upon an empirical estimate of the invariant distribution $f(K,\theta)$ of capital stock and productivity to obtain an evenly spaced grid of 50×50 points in the $K\times\theta$ space. Then we draw initial conditions (K_{-1},θ_{-1}) in that probability distribution of the economy with shocks to compute a 1500 periods deterministic transition to the non stochastic steady state.[3] These paths are denoted $\left\{C^B_{K_{-1},\theta_{-1}}(t), N^B_{K_{-1},\theta_{-1}}(t)\right\}_{t=0}^{1499}$.

We proceed in a similar way for the economy with shocks. Specifically, this amounts to simulating 1000 stochastic paths starting from the same initial conditions from which no-uncertainty transition paths have been computed. These paths are denoted correspondingly $\left\{C^A_{K_{-1},\theta_{-1}}(t), N^A_{K_{-1},\theta_{-1}}(t)\right\}_{t=0}^{1499}$.

[3]We extensively compute transition paths for all the cells of the 50 by 50 (K,θ) matrix, and then weight the utility of each of these paths with the density of its initial conditions.

We then evaluate $\mathcal{C}$, our comprehensive measure, using equation (35). We also compute the measure $\ell = \frac{1}{2}\sigma_z^2$ that corresponds to a non structural evaluation, with $\sigma_z^2 = E\left[\left(C^A - E(C^A)\right)^2\right]$.

4 Quantitative Findings

The choice of parameter values used in the simulation of the model is given in Table 1. All these values are standard in the literature. Note that we assume full depreciation to obtain an analytical solution. The disutility of labor parameter γ is set such that worked hours represent 20% of the time endowment at the non stochastic steady state.

Table 1: Parameters

Preferences		
Subjective discount factor	β	0.99
Disutility of labor	γ	3.9712
Technology		
Capital elasticity	α	0.42
Depreciation rate	δ	1
Shock process		
Serial Correlation of Tech. shock	ρ	0.95
Std. dev. of innovation	σ	0.01

Table 2 reports the welfare results.

Table 2: Welfare Cost of Fluctuations

	ℓ	$\mathcal{C}$
Walrasian economy	0.14%	0.14%
Nonclearing markets	0.16%	0.63%

First, notice that in the walrasian case, the comprehensive welfare cost of fluctuations $\mathcal{C}$ is small, and equal to the non structural measure ℓ. This result holds because the model is log-linear and shocks are log-normal. Second, the introduction of price rigidities sharply modifies the evaluation of the welfare cost of fluctuations, as $\mathcal{C}$ is now four time bigger. With low risk aversion and small shocks ($\sigma_\varepsilon = 0.01$), we measure the welfare cost of fluctuations to be about 0.66% of deterministic steady state consumption. In the nonclearing market economy, the dominant effects are the underutilization of labor and the waste of intermediate goods. In every period, worked hours are lower than what they would have been had the real wage been flexible. Therefore, capital is less productive, accumulation is lower, and output and consumption are lower than in a walrasian economy. On top of that, there is a waste of intermediate goods when productivity is lower than expected, as the final good firm does not buy all the production at the preset price. Once shocks are shut down, the allocation of the preset price economy coincides with the flexible economy one: the value of capital increases given that time is now efficiently allocated between work and leisure, and no intermediate goods are wasted. Therefore, the welfare cost of fluctuations is large. This cost is not properly measured by the non structural measure ℓ, as this measure does not take into account the increase in the mean of consumption that is associated with stabilization. The $\mathcal{C}$ measure, taking into account nonlinearities (the "min" function in this simple model) and computing the transition from the stochastic steady-state to the deterministic one, gives a comprehensive measure of this cost.

5 Concluding remarks

When a structural measure of the welfare cost of fluctuations is adopted, this cost appears to be non trivial in economies where allocations are inefficient, and where those inefficiencies are magnified by shocks. In the (admittedly specific) example of this paper, the comprehensive measure can be almost one order of magnitude larger than in a walrasian model, although it stays modest in our model economy. Working out the value of the comprehensive measure in more realistic environments is a potentially fruitful route, that we leave for further research.

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