Some Inference Perils of Imposing a Taylor Rule

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Abstract

The way monetary policy is conducted is a key element in New Keynesian models, and crucially determines allocations' properties. We show that assuming monetary authorities follow a Taylor rule may bias estimation of New Keynesian type models for two reasons. The first one is theoretically trivial, and is a standard misspecification bias that occurs if the actual conduct of policy does not follow the model specified Taylor rule. The second one is more subtle, and we refer to it as a determinacy bias. It occurs when wrongly assuming a Taylor rule restricts the set of admissible model deep parameters when one requires the equilibrium to be determinate, as is almost always the case in the applied literature. Using US data, we show that the determinacy bias is a serious problem in small scale New Keynesian models, as the slope of Phillips curve is biased upwards. The misspecification bias is a serious problem when estimating a medium-scale model, as it affects the contribution of the various shocks to macroeconomic fluctuations. We propose an alternative agnostic specification of the policy rule that is immune to both misspecification and determinacy biases.

Key Words: Taylor rule, DSGE estimation, New Keynesian model
JEL Class.: E31, E32, E47, C51

Introduction

It is well-known but sometimes forgotten that assuming nominal rigidities does not put many restrictions on real equilibrium allocations. What is relevant in the combination of sticky prices *and* monetary policy. In a way, it is for that very reason that nominal

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rigidities have been introduced in macroeconomic analysis of monetary policy. A perfect illustration of this importance of monetary policy is the canonical three-equation New Keynesian model with only demand shocks; model in which monetary policy can replicate the flexible price real allocations when it responds with infinite aggressiveness to inflation fluctuations. The specification of the monetary policy rule is therefore key in the derivation of the model equilibrium allocations properties. Given the importance of the monetary policy rule, it is surprising that little attention is paid to its specification in the applied macroeconomic literature, when estimating New Keynesian DSGE (Dynamic Stochastic General Equilibrium) models, *as if it should not matter for the model estimation*. An alternative justification for using one particular Taylor rule is that *one needs to assume something*. We contradict this argument as one may assume only that the instrument is a function of the minimal state space of the economy.

In this paper, we show that specifying the conduct of monetary policy as following a Taylor rule does seriously bias model deep parameters estimates as well as our understanding of macroeconomic fluctuations. Biases come from two different sources that we coin misspecification bias and determinacy bias. We show the existence of these two biases in simple abstract economies, and then show that they do materialise when estimating New Keynesian type models on US data. Small models, which are more prone to indeterminacy, suffer from the determinacy bias. We will show that this leads to an overestimation of the Phillips curve slope. We will then show that medium scale models suffer from the misspecification bias. To avoid such biases, we propose an alternative and agnostic specification of the conduct of monetary policy (a state rule) that nests the Taylor rule specification and that is immune to both types of biases.

Taylor rule(s)

There is a large applied literature on estimated monetary DSGE models, from the early work of Rotemberg and Woodford [1997] to Ireland [2004], Christiano, Eichenbaum, and Evans [2005] and Smets and Wouters [2007], and the recent the Federal Reserve Bank of New York's DSGE model (Negro, Gleich, Lee, Nallamotu, and Sengupta [2022]). Models in this literature feature a version of the Taylor rule that follows Taylor [1993b]. Let us briefly describe the wealth of specifications used in the literature. This is relevant as we will show later that differences in specifications can lead to important differences in allocation properties.

The original Taylor's [1993b] rule is (dropping constants)

$$i_t = \pi_t + .5\widehat{y}_t + .5\pi_{t-2},$$

where i is the federal fund rate, π is the rate of inflation and $\widehat{y}$ is an output gap. J. Taylor introduces this rule as "*a hypothetical but representative policy rule [... that] closely* *approximates Federal Reserve policy [...]*", but does analyses it in a full DSGE models.[1] The rule is not is not meant to describe the actual implementation of monetary policy, but rather to closely track the trajectory of the nominal interest rate.

To the best of our knowledge, the first micro-founded and estimated New Keynesian model with an interest rule is Rotemberg and Woodford [1997]. The authors *"assume that recent U.S. monetary policy may be described by a feedback rule for the federal funds rate of the form"*

$$i_t = i^\star + \sum_{k=1}^{n_i} \mu_k (i_{t-k} - i^\star) + \sum_{k=1}^{n_\pi} \phi_k (\pi_{t-k} - \pi^\star) + \sum_{k=1}^{n_y} \theta_k y_{t-k} + \epsilon_t.$$

i is the Federal funds rate, π inflation, y the percentage deviation of real GDP from trend, and $i^\star$ and $\pi^\star$ are long run "targets". Rotemberg and Woodford [1997] justify this rule by following " *the monetary-policy "reaction function" literature, especially Taylor [1993b]*". The rule is estimated from a VAR in a first stage, and the rest of the model is estimated, and some counterfactual simulations are done with the Taylor rule

$$i_t = \theta_\pi \pi_t + \theta_y y_t.$$

Note that this is not exactly the specification of Taylor [1993b] we reported before. A very similar exercise is performed by Christiano, Eichenbaum, and Evans [2005] in a richer model with capital, and counterfactual simulations are performed with a Taylor rule specified as

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi E_{t-1}[\pi_{t+1}] + \phi_y y_t\right),$$

There is no motivation for that particular specification but "*It is commonplace in the literature to represent monetary policy as a parsimonious Taylor rule.*" Note that in their specification inflation enters as $E_{t-1}[\pi_{t+1}]$.

The first paper we know that jointly estimated Taylor rule and other equations of a DSGE is Ireland [2004], who chooses the specification:

$$i_t = \rho_i i_{t-1} + \rho_y y_{t-1} + \rho_\pi \pi_{t-1} + \varepsilon_{it}.$$

The justification for this specification is that this rule "*generalizes Taylor's rule, however, by adding a term involving the lagged interest rate: when ρ_i is nonzero, the interest rate adjustment to output and inflation occurs gradually over time.*" Note that the output gap enters here with a lag.

The classic reference for estimated DSGE paper is Smets and Wouters [2007], in which the specification is

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \phi_{\Delta y}\left(y_t - y_{t-1}\right),$$

[1] Taylor [1993a] adds a Taylor rule to a large econometric model that is not microfounded and estimated equation by equation

This rule is introduced in the following way: "*Finally, the model is closed by adding the [above] empirical monetary policy reaction function [...] The monetary authorities follow a generalized Taylor rule.*" Note the addition of the growth rate of the output gap to the rule.

Finally, Negro, Gleich, Lee, Nallamotu, and Sengupta [2022] represents the latest vintage of New Keynesian DSGE models, and it features a Taylor rule of the type

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\psi_1(\pi_t - \pi_t^\star) + \psi_2 y_t\right) + \psi_3\left(y_t - y_{t-1}\right) + \varepsilon_t^i,$$

Tis rule is introduced by "*the monetary authority follows a generalized policy feedback rule.*" Note here that a moving inflation target $\pi_t^\star$ has been included in the rule.

The point that we want to make here is that the choice of a particular specification of the Taylor rule is not justified as a better description if the *actual implementation* of monetary policy by the Fed. We speculate that the choice is driven by a trade-off between giving the model a good fit and not deviating too much from Taylor original rule. We will show that the specification choice is unfortunately likely to create biases in the deep parameters estimation.

The importance of the Taylor rule specification

Let us showcase the crucial role of the Taylor rule specification in shaping equilibrium allocations. Consider the following (reasonably parametrized) three-equation New Keynesian model:

$$y_t = E_t[y_{t+1}] - (i_t - E_t[\pi_{t+1}]) + d_t, \tag{1}$$

$$\pi_t = 0.99\ E_t[\pi_{t+1}] + 0.1\ y_t + \mu_t, \tag{2}$$

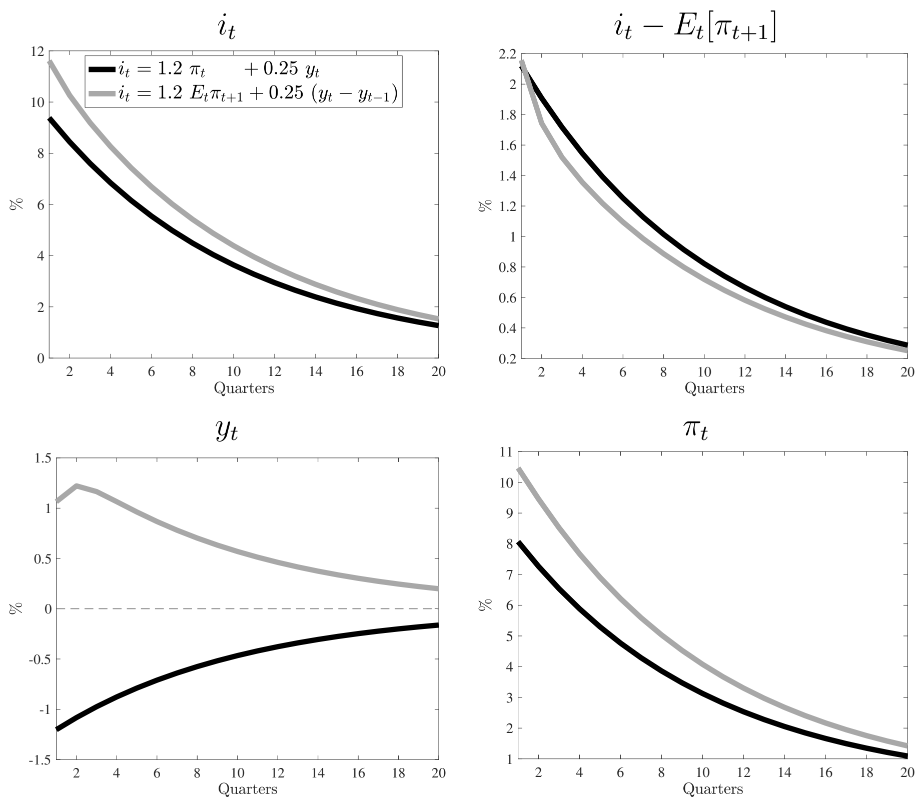
where d and μ are autoregressive shocks with persistence 0.9 and unit innovation variance. We close the model with one of the two following Taylor rules:

$$i_t = 1.2\ \pi_t + 0.25\ y_t, \tag{3}$$

$$i_t = 1.2\ E_t\pi_{t+1} + 0.25\ (y_t - y_{t-1}). \tag{4}$$

These two "Taylor rules" are consistent with the narrative that monetary authorities set the nominal interest rate in reaction to inflation (current or expected) and the output gap (level or growth rate). We consider the response to the following combination of shocks: an innovation of 1 to d and 2 to μ. The impulse responses to that combination of shocks are represented on Figure 1. It is striking to see that although "instruments" (real or nominal interest rates) respond in qualitatively (and to a large extent quantitative) similarly with the two policy rules, qualitative responses of output and inflation are totally different: the same set of shocks does increase output with one rule and decrease it with the other one; inflation and output co-move with one rule, while they move in opposite direction with the other rule. This example (in particular the set of shocks) is admittedly peculiar, and has no other virtue than illustrating the qualitative dependance of allocations properties to the specification of the Taylor rule.

Figure 1: Impulse response to a combination of d and μ shocks



Notes: The IRF are derived from the model (1), (1) and Taylor rule 3) or(4). The shock is a combination of d (size 1) and μ (size 2).

State Policy Rule

Throughout the paper, we will contrast Taylor rules with "state rules". We define a state rule as a rule in which the policy instrument is a function of the minimal state space of the economy in the spirit of McCallum [1983]. It corresponds to what Svensson and Woodford [2005] call an explicit instrument rule. As we will show, the advantage is that the state rule we will use will guarantee determinacy while staying agnostic about the actual implementation of monetary policy. Furthermore, we will prove that such a rule is encompassing all the other rules. An argument against such a state rule is that it supposes that the central bank observes the full state space of the economy, as it is discussed for example in Giannoni and Woodford [2003]. That would be a concern if we were to identify the optimal policy. We do not look for optimal monetary policy, but we simply want to estimate the actual policy, so that potential restrictions in the information set of the Fed will be estimated by our state rule. One may argue that it is more efficient for the estimation to *ex ante* restrict the state space that policy maps into the instrument, using priors coming from the study of the Fed decision process. It is true, but that should be explicitly justified by an in-depth analysis of the actual implementation of monetary policy. As we have shown above, this is not what is done when assuming one or another specification of the Taylor rule. Our state rule is totally agnostic about the implementation of policy, and therefore also agnostic about the information set of the policy maker. We will show that if wrongly used, Taylor rules create misspecification and determinacy biases in the estimation of all the model deep parameters. Misspecification bias is hardly a surprising result in theory. Our contribution is to show that it matters big time, and changes our account of macroeconomic fluctuations, as seen through the lens of a medium scale New Keynesian DSGE. Determinacy bias occurs when assuming wrongly a Taylor rule restricts the set of admissible model deep parameters when one requires the equilibrium to be determinate, as almost always done in the applied literature. We show that the determinacy bias is pervasive when estimating small models.

Outline

Section 1 will theoretically illustrate the consequences of the dependance of allocations and parameters estimates to the specification of the monetary policy rule. The empirical relevance of problem when estimating small scale (Section 2) and medium scale (Section 3) models.

1 Determinacy Bias and Misspecification Bias

In this section, we show that the use of a Taylor rule type of policy rule can lead to determinacy and misspecification biases in estimation. We also show that they can be replaced by state rules that are not subject to both types of bias.

1.1 Determinacy Bias

We propose here a simple abstract model in which we can illustrate how the use of a "Taylor" type of policy rule can generate a determinacy bias.

Model

Let's consider the following forward looking equation:

$$y_t = \alpha E_t y_{t+1} + \beta i_t + s_t, \tag{5}$$

with $0 \leq \alpha < 1$ and for simplicity $\beta > 0$. y is the variable of interest, that is set by the private sector. y also depends on a policy instrument i that is controlled by a policy maker. Finally, s is an autoregressive shock

$$s_t = \rho s_{t-1} + \varepsilon_t, \tag{6}$$

with $0 \leq \rho < 1$ and ε is iid with unit variance. We do not specify what is the objective of the policy maker, and in that sense, we are not looking for optimal rules. We nevertheless restrict the policy maker choice to rules that guaranty determinacy of the solution, as this is what the applied DSGE literature is doing.[2].

As previously written, i is an *instrument*, a policy variable that helps controlling y in order to reach some objective that we . The policy maker can choose two types of policy rules. The first type makes the instrument reacting to some of the endogenous variables of the current period (an example is the Taylor rule). We coin it a feedback rule. In this simple example, the feedback ("ϕ*eedback*") rule will be

$$i_t = \phi y_t. \tag{7}$$

The second type makes the instrument reacting to the state of the economy. Given the restriction to determinate outcomes, the state space will be the minimal set of state variables. As alluded to in the introduction, we coin such a rule a state rule. In this simple example, the only state variable is the shock s_t. and the state ("σ*tate*")- rule will be

$$i_t = \sigma s_t + \nu_t. \tag{8}$$

Note that in this example, the state of the economy is unidimensional, and both rules map one variable into the instrument. We show below that the feedback rule is nevertheless restrictive, which can cause bias in estimation.

Solution with a feedback rule

In the case of a feedback rule, plugging (7) into (5) and substituting forward gives

$$y_t = \frac{1}{1-\beta\phi}\left(\sum_{j=0}^{\infty}\left(\frac{\alpha\rho}{1-\beta\phi}\right)^j\right) s_t \tag{9}$$

[2] There are a few exceptions to that statement, for example, Lubik and Schorfheide [2004], Beyer and Farmer [2007] and Bianchi and Nicolò [2021]

The condition for this sum to converge for any admissible persistence parameter ρ is $\left|\frac{\alpha}{1-\beta\phi}\right| < 1$. In that case, the solution of the model will be determinate. This condition can be written as a condition on the policy parameter ϕ, and is given by

$$\phi \notin \left]\frac{1-\alpha}{\beta}, \frac{1+\alpha}{\beta}\right[. \tag{10}$$

This condition echoes the "Taylor principle" condition in New Keynesian models. When the condition (10) is satisfied, the solution of the model under a feedback rule is given by

$$y_t = \frac{1}{1-\beta\phi-\alpha\rho}\, s_t. \tag{11}$$

Solution with a state rule

In the case of a state rule, plugging (8) into (5) and substituting forward gives

$$y_t = (1+\beta\sigma)\left(\sum_{j=0}^{\infty}(\alpha\rho)^j\right) s_t \tag{12}$$

In that case, because α is assumed to be in the unit interval, the sum converges for any admissible ρ and *any* policy choice σ, and the solution of the model under a state rule is given by

$$y_t = \frac{1+\beta\sigma}{1-\alpha\rho}\, s_t \tag{13}$$

Equivalent rules

Is the choice of the rule – in other words the description of actual implementation of policy – irrelevant? Here we show that for any feedback rule for which a determinate solution exists, there exist an equivalent – meaning spanning the same allocations – state rule, but that the converse is not true.

Consider first a feedback rule model with parameter ϕ such that the solution is determinate (condition (10) is satisfied). Comparing (11) and (13), we can see that a state rule with parameter σ^E generates the same allocations that a feedback rule under the necessary condition:

$$\sigma^E = \frac{\phi}{1-\beta\phi-\alpha\rho}.$$

As the equilibrium with state rules is always determinate, the above necessary condition is also sufficient.

Consider now a state rule model with parameter σ. Comparing again solutions (11) and (13), a necessary condition for a feedback rule with parameter ϕ^E to generates the same allocations that a state rule with parameter σ) is that

$$\phi^E = \frac{\sigma(1-\alpha\rho)}{1+\beta\sigma}.$$

But that condition may not be sufficient, as the equilibrium with feedback rule ϕ^E may not ne determinate. If $\sigma > -\frac{1+\alpha}{2\alpha\beta}$, then ϕ^E will satisfy the determinacy condition (10). But if $\sigma < -\frac{1+\alpha}{2\alpha\beta}$, there is no determinate feedback model that can reproduce the state rule allocations.

These results show that under the assumption that the equilibrium is determinate, as for example almost always the case in monetary DSGE models, assuming a feedback rule is putting restrictions on allocations while assuming a state rule does not. If the Data Generating Process (DGP) features a state rule, then assuming a feedback rule might biased the estimation if ϕ^E does not satisfy the determinacy condition (10), while the converse is not true. Below, we illustrate this in a very specific case, and then in a simulation exercise.

Implications for estimation, an analytical case:

In order to obtain analytical results, we make here some rather stark assumption about what is known and observed by the econometrician. We will relax later these assumptions in a simulation exercice. Let us assume that the DGP is the feedback rule model, that β, ρ, ϕ and σ^E are known to the econometrician and that only y is observable. The econometrician aims at estimating the only unknown parameter α. In such a case, the maximum likelihood estimator of α can be obtained by matching the model unconditional variance of y with its sample counterpart. With an infinite amount of data, the model unconditional variance of y is equal (using equation (11)) to

$$V^{\phi}(\alpha) = \frac{1}{(1-\beta\phi-\alpha\rho)^2}\,\frac{1}{1-\rho^2}.$$

Here $V^{\phi}(\alpha)$ denotes the variance of y in the model with parameter α and a feedback rule with parameter ϕ. Assume that the econometrician wrongly believes that the DGP is a state rule model, so that the variance of y writes (using equation (13)):

$$V^{\sigma}(\alpha) = \left(\frac{1+\beta\sigma^E}{1-\alpha\rho}\right)^2 \frac{1}{1-\rho^2}.$$

An estimator of α is then given by $\widehat{\alpha}$ that solves

$$V^{\sigma}(\widehat{\alpha}) = V^{\phi}(\alpha) \tag{14}$$

Equation (14) has two solutions in $\widehat{\alpha}$, that are α and $2/\rho-\alpha$. The second one is greater than one. Therefore, selecting the solution that gives determinacy under a state rule (i.e. imposing $|\widehat{\alpha}| < 1$) always provides an unbiased estimate of α. Wrongly assuming that the economy has a state policy rule is of no consequence for the estimation of α.

Let us now assume the reverse, i.e. that the DGP is a model with a state rule. Again, the econometrician knows all the parameters, including the equivalent feedback rule parameter ϕ^E, but has to estimate α observing only y and assuming that the DGP is the feedback rule model. This can be done again by matching the variance of y, In this case, the variance of y is is asymptotically given by

$$V^\sigma(\alpha) = \left(\frac{1+\beta\sigma}{1-\alpha\rho}\right)^2 \frac{1}{1-\rho^2}.$$

According to the econometrician DGP, the variance of y is

$$V^\phi(\alpha) = \frac{1}{(1-\beta\phi^E-\alpha\rho)^2}\,\frac{1}{1-\rho^2}$$

An estimator of α is then given by $\widehat{\alpha}$ that solves

$$V^\phi(\widehat{\alpha}) = V^\sigma(\alpha) \tag{15}$$

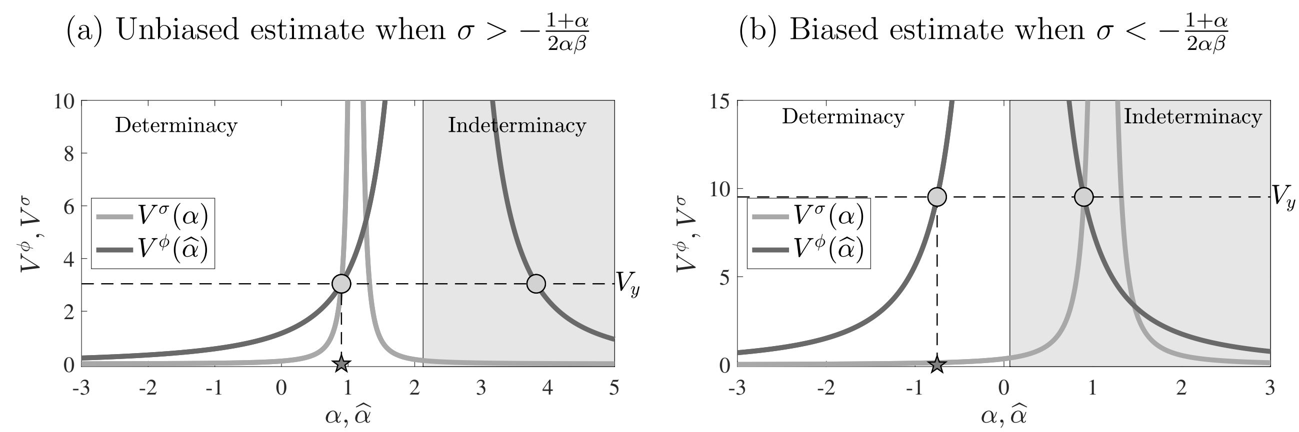
Again, this equation has two solutions in $\widehat{\alpha}$, that are α and $\frac{(\beta\sigma-1)}{1+\beta\sigma}\ \alpha + \frac{2}{\rho(1+\beta\sigma)}$, one under which the model is determinate and one under which it is not.

Imposing that the model is determinate under $\widehat{\alpha}$ allows to select among the two possible values of $\widehat{\alpha}$. Recall that a feedback model with equivalent ϕ^E is determinate if and only if $\sigma > \frac{1+\alpha}{2\alpha\beta}$. By the very definition of ϕ^E (equivalent ϕ), the model is in that case determinate for $\widehat{\alpha} = \alpha$, and not for $\widehat{\alpha} = \frac{(\beta\sigma-1)}{1+\beta\sigma}\ \alpha + \frac{2}{\rho(1+\beta\sigma)}$. In this case, α is estimated without bias. But if parameters are in the region $\sigma > \frac{1+\alpha}{2\alpha\beta}$, then imposing determinacy leads to the selection of the biased value $\widehat{\alpha} = \frac{(\beta\sigma-1)}{1+\beta\sigma}\ \alpha + \frac{2}{\rho(1+\beta\sigma)}$. What happens is that the restriction for determinacy "forces" the econometrician to choose the wrong value of α. Note that in this extreme case, this comes at no cost in terms of likelihood.

The above result can be illustrated graphically. Assume $\alpha = .9$, $\beta = .2$ and $\rho = .9$. We consider two values for σ. In the first case, $\sigma = -4.3 > -\frac{1+\alpha}{2\alpha\beta}$, so that the feedback model with equivalent ϕ^E is determinate. In the second case, $\sigma = -6.3 < -\frac{1+\alpha}{2\alpha\beta}$, so that the feedback model with equivalent ϕ^E is indeterminate.

In the first case, $\sigma > -\frac{1+\alpha}{2\alpha\beta}$. It corresponds to panel (a) in Figure 2. On this Figure, the light gray line plots the variance of y denoted $V^\sigma(\alpha)$ as a function of α. Only one point on this line is relevant as $\alpha = .9$. This point is the dark dot. It corresponds to the observed variance of y. The dark line plots $V^\phi(\widehat{\alpha})$. It represents the variance of y under the assumption that the DGP is has a feedback rule and when the estimated α is $\widehat{\alpha}$. The black vertical line separate the plane into a white zone in which $\widehat{\alpha}$ is such that the model is determinate, and a grey zone in which it is indeterminate. The two candidates for the estimation of α are the two values of $\widehat{\alpha}$ for which $V^\phi(\widehat{\alpha}) = V^\sigma(\alpha)$. They correspond to the two dots at the intersection of the horizontal dashed line whose ordinate is $V^\sigma(\alpha)$ and the variance if y when the model is a feedback rule one, which is the dark line $V^\phi(\widehat{\alpha})$. In this configuration of parameters, selecting the determinate solution leads to the unbiased estimate $\widehat{\alpha} = \alpha$ (the star on the figure). Panel (b) of Figure 2 corresponds to the case in which the unbiased estimate is not chosen because it is in the indeterminacy zone, so that α is estimated at the star (in the determinacy zone), which corresponds to $\widehat{\alpha} = -0.75$ instead of 0.9.

Figure 2: Determinacy Bias in the Estimation of α



Notes: See main text for detailed explanation of this Figure.

Implications for Estimation, A Monte Carlo Illustration

Le us now assume that the econometrician observes y and i and is estimating all the parameters of the model except for the variance of shocks which are assumed to be known. In that case again, the estimation is constrained to determinate models. As we cannot analytically derive the determinacy bias, we provide illustrative simulations.

Assume a state rule model model given by equations (5), (6) and (8). Let's assume $\alpha = .99$, $\beta = .2$ and $\rho = .9$. We consider two values for σ. In the first case, $\sigma = -4.3 > -\frac{1+\alpha}{2\alpha\beta}$, the feedback rule model with equivalent ϕ^E is determinate. In the second case, $\sigma = -6.3 < -\frac{1+\alpha}{2\alpha\beta}$, the feedback rule model with equivalent ϕ^E is indeterminate. To avoid stochastic singularity, we assume that there is a shock ν to the policy rule that is is iid with unit variance.

For the two state models, we run 100,000 simulations of length 1,000, and estimate by Maximum Likelihood a feedback rule model on each of those 100,000 samples. Results from this Monte Carlo exercise are presented in Table 1. As we can see in this table, there is no bias in estimating a feedback rule model when the equivalent feedback rule model is determinate, but α is seriously biased when the equivalent feedback rule model is indeterminate. Biasing α is the only way for a feedback rule model to fit the data while ensuring determinacy. As shown on Figure 3, imposing a feedback rule in the estimation can seriously bias the impulse response to a s shock.

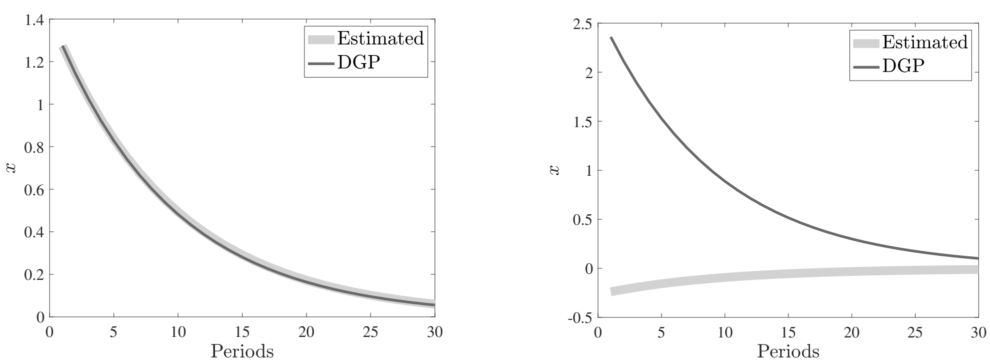
Table 1: Estimation of a feedback rule model when the DGP is a state rule model

When the equivalent feedback model is determinate $\sigma > -\frac{1+\alpha}{2\alpha\beta}$					
	α	β	ρ	σ	ϕ^E
DGP	.99	.2	.9	-4.31	-3.38
Estimation	.91	.18	.90	–	-3.35
	(.18)	(.05)	(.01)	–	(.02)
When the equivalent feedback model is not determinate $\sigma < -\frac{1+\alpha}{2\alpha\beta}$					
	α	β	ρ	σ	ϕ^E
DGP	.99	.2	.9	-6.31	2.63
Estimation	-.06	.24	.90	–	2.61
	(.09)	(.03)	(.01)	–	(.01)

Notes: in this table we report the mean of the point estimates over the 100,000 simulations of length 1,000, with the standard deviation of the point estimates over the 100,000 simulations between parenthesis.

Figure 3: DGP and estimated impulse responses to a s shock

Determinate equivalent feedback model   Indeterminate equivalent feedback model



Notes: on the left panel, the DGP is a state rule model with $\sigma > -\frac{1+\alpha}{2\alpha\beta}$, so that the equivalent feedback rule model is determinate. On the right panel, the DGP is a state rule model with $\sigma < -\frac{1+\alpha}{2\alpha\beta}$, so that the equivalent feedback rule model is indeterminate. The values of the parameters are given in Table 1. Estimated IRF are computed with the mean value of the estimated coefficients over the 100,000 simulations of length 1,000.

1.2 Misspecification Bias

A second type of bias (misspecification bias) may occur if one wrongly puts restrictions on the policy rule by assuming a feedback rule, whereas a state rule would never face such a bias. This bias is we believe well understood, and we illustrate it using an abstract simple model.

Model

The model has dynamics and two shocks. The variable of interest y follows

$$y_t = \beta i_t + s_{1t} + \gamma s_{2t}, \tag{16}$$

where (s_1, s_2) are two iid unit variance shocks. i is the policy variable that helps controlling y. The policy maker can again choose two type of policy rules, a feedback rule or a state rule. The feedback rule is

$$i_t = \phi y_t + \nu_t, \tag{17}$$

and the state rule is

$$i_t = \sigma_1 s_{1t} + \sigma_2 s_{2t} + \nu_t. \tag{18}$$

ν_t is a iid shock with unit variance. Note that in this example, the state of the economy is bi-dimensional. The state rule rule maps the two state variables into the instrument, while the feedback rule maps a one-dimensional space into the instrument. It is no surprise that if the DGP has a state rule, the feedback rule model will be generically misspecified

In the case of the feedback rule model, plugging (17) into (16) gives the solution

$$y_t = \frac{1}{1-\beta\phi} s_{1t} + \frac{\gamma}{1-\beta\phi} s_{2t} + \beta\nu_t. \tag{19}$$

In the case if the state rule model, plugging (18) into (16) gives the solution

$$y_t = (1+\beta\sigma_1)s_{1t} + (\gamma + \beta\sigma_2)s_{2t} + \beta\nu_t \tag{20}$$

Estimating γ

Assume that γ is the unknown deep parameters of interest, that β and the variances of s_1 and s_2 are known to the econometrician and that y, i, s_1 and s_2 are observed. When the true model is known, the econometrician can identify the model parameters (γ, ϕ) or $(\gamma, \sigma_1, \sigma_2)$ in the following way. The policy parameters ϕ or σ_1 and σ_2 are obtained by estimating equations (17) or (18). Then, estimating the solution equations (19) or (20) gives the multiplier $\mu_2 = \frac{\partial x}{\partial s_2}$, that allows for the identification of γ.

How is the estimation affected by a misspecification of the policy rule? Let us first assume that the DGP is the feedback rule model. If the econometrician wrongly assumes that the policy is the state one, that will be inconsequential, as the state rule nests the feedback one. Indeed, the econometrician will believe that the multiplier is

$$\mu_2 = \gamma + \beta\phi_2$$

She will then first estimate equation (18) to obtain $\widehat{\sigma}_1$ and $\widehat{\sigma}_2$, and those estimators will be asymptotically equal to $\widehat{\sigma}_1 = \phi(1-\beta\phi)^{-1}$ and $\widehat{\sigma}_2 = \gamma\phi(1-\beta\phi)^{-1}$ (using the solution of the true DGP (19)). In a second step, she will use the misspecified model to back out γ from

$$\widehat{\mu}_2 = \widehat{\gamma} + \beta\widehat{\sigma_2},$$

so that

$$\widehat{\gamma} = \widehat{\mu}_2 - \beta\widehat{\sigma_2}.$$

Because the DGP has a feedback rule, we know that $\widehat{\mu}_2 = \gamma(1-\beta\phi)$. Given the value of $\widehat{\sigma_2}$, we obtain

$$\widehat{\gamma} = \gamma(1-\beta\phi) - \beta\gamma\phi(1-\beta\phi)^{-1} = \gamma.$$

As we said, because the misspecified state rule encompasses the feedback rule, the estimation of γ is unbiased.

Let us now assume that the DGP is the state rule model, and that the econometrician wrongly believes that it is the feedback rule one. The econometrician will first estimate the feedback rule (17), which amount to solve

$$\min_{\phi} E\left[((\phi-\sigma_1)s_{1t} + (\phi-\sigma_2)s_{2t})^2\right],$$

and the solution to this estimation problem is

$$\widehat{\phi} = \frac{1}{2}(\sigma_2 + \sigma_2).$$

γ will then be identified, under the belief that the model has a feedback rule policy, by solving

$$\widehat{\mu}_2 = \widehat{\gamma}(1-\beta\widehat{\phi})^{-1}$$

where $\widehat{\mu}_2 = \gamma + \beta\sigma_2$ because the DGP is a state rule model. Asymptotically, the estimator of γ will therefore be

$$\widehat{\gamma} = \left(1 - \frac{\beta}{2}(\sigma_1+\sigma_2)\right)\gamma + \beta\sigma_2\left(1 - \frac{\beta}{2}(\sigma_1+\sigma_2)\right)$$

Misspecification will therefore create a bias in the estimation of γ, except in the non-generic case in which $\sigma_1 = \frac{(1-\gamma))}{\beta(1+\gamma)}$ and $\sigma_2 = \frac{\gamma(1-\gamma))}{\beta(1+\gamma)}$.

2 Small Scale New Keynesian Models

We have shown in the previous section that Taylor rules (and more generally feedback rules) may create determinacy and misspecification biases. In this section, we show that these biases do matter in practice when estimating a three-equation New Keynesian model or various extension of that model.

2.1 Three Equation Model with Taylor or State Rule

We start by estimating the following prototypical linearised New Keynesian model as in Woodford (2003) and Gali (2015), where the first two equations are given by:

$$\begin{aligned} y_t &= \alpha_y E_t y_{t+1} - \frac{1}{\gamma}(i_t - E_t \pi_{t+1}) + d_t \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa y_t + \mu_t \end{aligned}$$

The first of these is the Euler equation, and the second is the New Keynesian Phillips curve. We allow for "discounting" in the Euler equation[3] ($|\alpha_y| < 1$) for a reason that will be clear later in this section, but α_y can be arbitrarily close to one. β is the household's subjective discount rate, γ is the coefficient of relative risk aversion and κ is the slope of the Phillips curve. d_t and μ_t are "demand" and "supply" shocks respectively[4], and both follow AR(1) processes of the form

$$d_t = \rho_d d_{t-1} + \varepsilon_t^d,$$

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu,$$

with respective standard deviation of innovations s_d and s_μ. The third equation of the model is a monetary policy rule that sets the nominal interest rate. One possibility we will explore is the standard Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + \nu_t,$$

Note that this rule can be rewritten without loss of generality as a real interest rule:

$$i_t - E_t \pi_{t+1} = E_t \pi_{t+1} + \phi_\pi \pi_t + \phi_y y_t + \nu_t,$$

where ν_t is a monetary shock that follows an AR(1) process:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,$$

with standard deviation of innovations s_ν. The alternative rule we will consider is a state rule that maps the state of the economy into the real interest rate. Note that the minimal model state consists here of the three shocks, as the model is purely forward looking. The state rule is

$$i_t - E_t \pi_{t+1} = \sigma_d d_t + \sigma_\mu \mu_t + \nu_t$$

[3]See McKay, Nakamura, and Steinsson [2016], Farhi and Werning [2019], Gabaix [2020] or Beaudry, Hou, and Portier [2020] for microfoundations.

[4]The two shocks can be derived from shocks to the household discount factor and shocks to the elasticity of substitution between varieties if preferences are Dixit and Stiglitz [1977]

This rule is not meant to describe the actual implementation of monetary policy. As discussed in the previous section, for any feedback policy restricted to give a determinate equilibrium, there exist a state rule that replicates the same equilibrium allocations. The assumption of a discounted Euler equation $|\alpha_y| < 1$ makes the equilibrium determinate with the real interest state rule for any parameters σ_d and σ_μ. Indeed, with such a state policy (and this is the reason why we have chosen a stat rule for the real interest rate), the model becomes recursive. Replacing in the Euler equation $i_t - E_t\pi_{t+1}$ by its expression in the state rule gives

$$y_t = \alpha_y E_t y_{t+1} + \left(1 - \frac{\sigma_d}{\gamma}\right) d_t - \frac{\sigma_\mu}{\gamma}\mu_t - \frac{1}{\gamma}\nu_t$$

which can be uniquely solved forward for any shocks persistence as long as $|\alpha_y < 1|$. Then the solution for y can be plugged into the Phillips curve, that can also be uniquely solved forward as $|\beta| < 1$.

As we said, the real interest state rule is an agnostic representation of monetary policy, that does not impose any restrictions on how monetary policy is conducted[5]. The way we are here thinking of monetary policy is as follows. The Euler equation and Phillips curve describe an hyper surface $\{y_t, \pi_t\}_{t=0}^{\infty}$. Given the shocks, the equilibrium allocation is one point on that hyper surface. One way to describe monetary policy is that it pins down that point, and an agnostic way of describing the selection of that point for estimation is the real interest rate state rule we have introduced. But an output state rule of the type $y_t = \sigma_d d_t + \sigma_\mu \mu_t + \nu_t$, estimated together with the Euler equation and the Phillips curve, would replicate the same allocations and equal deep parameters estimates.[6] We prefer to use the real interest rate state rule as the interpretation of the rule parameters σ_d and σ_d is more natural.

This first example of a purely forward New Keynesian model fits exactly the first abstract model presented in the previous section. In the Taylor rule, the nominal interest rate reacts to two endogenous variables plus the monetary policy shock. As there are exactly two other shocks, estimation with the Taylor rule will not be subject to misspecification as compared to an estimation with the state rule, but may be subject to indeterminacy bias. Note that the condition for determinacy in the Taylor rule model is a form of the well-known Taylor principle:

$$\Omega = \phi_y \left(\frac{1-\beta}{\kappa}\right) + \phi_\pi > 1 \tag{21}$$

In order to compare to the "traditional" New Keynesian model without Euler discounting, we will set $\alpha_y = 0.999$ such that there is quasi-no Euler discounting.

[5]This real interest rate rule is different from Holden's [2022] "real rate rules" defined as $i_t = r_t + \phi\pi_t$. This latter rule is a feedback rule shown to be "robust" in the sense that it avoids indeterminacy in many environments.

[6]This is true in theory, and we have checked that it is also true in practice.

2.2 Estimation

We estimate the model over the period 1959:I-2019:IV. The data series we use are the output gap from the CBO, the inflation rate measured as the log difference of the CPI, and finally the Federal Funds rate. The shadow Federal Funds rate from Wu and Xia [2016] is used from 2009 onwards - the period when the zero lower bound served as a binding constraint. We calibrate $\beta = 0.99$ and $\gamma = 1$ (log utility), both standard values, and then proceed to estimate the remaining parameters with Bayesian methods. We specify uniform priors for the state rule parameters, and standard prior distributions for all other parameters. Table 2 displays estimated parameters under the two rules.[7]

Table 2: Estimation Results, Simple Three Equation Model with a Taylor Rule or a State Rule

	Taylor R.	State R.		Taylor R.	State R.
κ	.678	.006	ρ_d	.962	.992
	(.0626)	(.0014)		(.0121)	(.0038)
ϕ_π	1.769	-	ρ_μ	.994	.371
	(.1741)	-		(.0031)	(.0317)
ϕ_y	-.014	-	ρ_ν	.457	.924
	(.0128)	-		(.0182)	(.0104)
σ_d	-	.971	s_d	.092	.102
		(.0145)		(.0129)	(.0104)
σ_μ	-	-.463	s_μ	.575	.324
	-	(.0657)		(.0535)	(.0235)
			s_ν	.811	.057
				(.0878)	(.0077)

Notes: Estimated model is the simple three-equation New Keynesian model with a Taylor rule. Posterior standard deviations are reported between parenthesis. Sample is 1959Q1-2019Q4.

Monetary policy rules parameters show that the Taylor rule is mainly reacting to inflation, while the state rule shows that the Fed increases the real interest rate in response to demand shocks and decreases it in response to supply shocks.

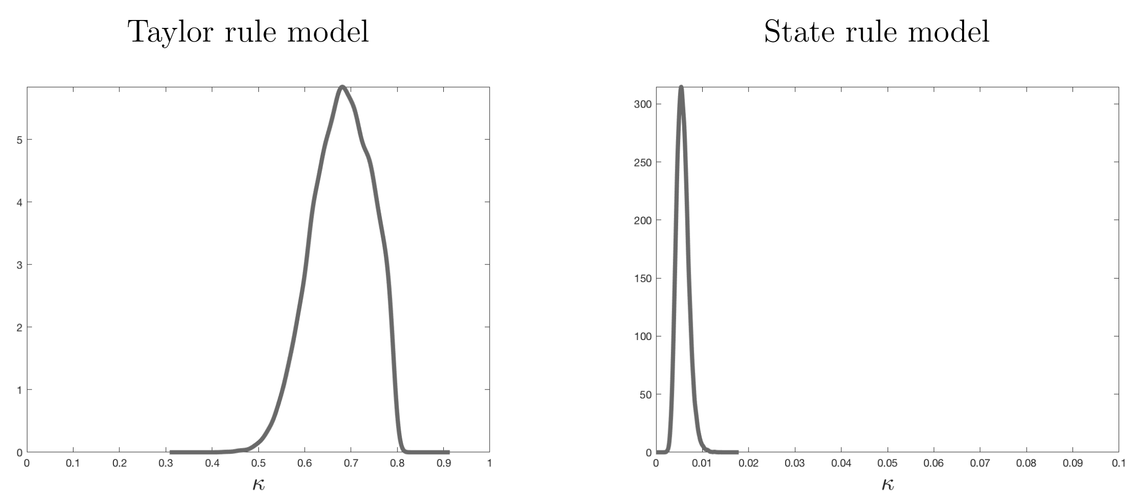
The most notable difference in parameter estimates is κ, the slope of the Phillips curve - 0.678 in the Taylor rule model at the posterior mean vs 0.006 in the state rule model.[8] Figure 4 compares the posterior distribution of κ in the two models. The

[7] In the appendix, Tables A.1 and A.2 present priors and full estimation results.

[8] A second difference is that supply shocks μ_t are estimated to be both much more persistent and to exhibit more variance in the Taylor rule model. Variance decompositions show that the demand shock is estimated to be the most important source of output fluctuations in the state rule model, while the supply shock is the primary shock for business cycles in the Taylor rule model. Since the model is highly stylised,

larger value in the Taylor rule model implies quite a steep estimated Phillips curve, while we estimate a very flat one in the state rule, with almost all the mass on the range between 0 and 0.015. Moreover, the estimate in the state rule model lies in the range typically estimated when using single-equation estimation of the Phillips curve. For example, Mavroeidis, Plagborg-Møller, and Stock [2014] conduct such estimates over a very large number of different specifications, and find that the median estimate across these is 0.004, which is very close to the posterior mean we estimate in the state rule model. Our estimate exactly coincides with that in Hazell, Herreño, Nakamura, and Steinsson [2022] for their baseline specification. The posterior mean in the Taylor rule model is implausibly large compared to the range of single-equation estimates in the literature. It is also in line with the estimated slope of Beaudry, Hou, and Portier [2020] who consider a Phillips curve augmented with a cost channel.

Figure 4: Comparison of the Posterior Distributions of the Phillips Curve Slope κ



Notes: this Figure compares the estimator posterior distributions of the Phillips curve slope parameter κ for a simple three-equation New Keynesian model, with a Taylor rule or a state rule. Sample is 1959Q1-2019Q4.

How can we explain such a difference between the two estimated Phillips curve slopes? The only extra restriction that the Taylor rule specification puts on the data, as compared to the state rule specification, is that Taylor rule parameters must guarantee determinacy. On the contrary, determinacy is always granted with the state rule model. These results suggest that we have here the determinacy bias, as defined in the previous section. This is most clearly understood by doing the following exercice. We take the

with many frictions and shocks omitted, a variance decomposition should not be taken too literally.

Table 3: Estimated and Implied Taylor Rule

	Taylor Rule	Implied Taylor Rule
ϕ_π	1.77	-0.24
ϕ_y	-0.01	0.68

Notes: Estimated model is the simple three-equation New Keynesian model with a state rule. Sample is 1959Q1-2019Q4.

state rule estimates and backup the implied Taylor rule, in the sense of the previous section. In words, the implied Taylor rule is the only candidate Taylor rule that, for the deep parameters estimates we have obtained with the (unrestricted) state rule, would produce the same equilibrium allocations.

Table 3 displays the implied Taylor rule at the posterior mean as well as the Taylor rule we obtain when estimating the model under a Taylor rule. The implied Taylor rule has a negative coefficient on inflation, which is a trivial violation of the Taylor principle. and would produce indeterminacy in the model with the Taylor rule. Thus, when we allow for a state rule that nests the Taylor rule, the data strongly favours parameter combinations that produce indeterminacy in a standard New Keynesian model. This suggests that the restriction of determinacy, which is implicitly imposed whenever the standard model is estimated, is substantially distortionary and has material impact on parameter estimates. This shows that for this simple forward looking model estimated on US data, the determinacy bias is indeed present.

Finally, we compare how well each model fits the data. To ensure a valid comparison, it is necessary that the prior integrates to one for all compared models. For state rule model, this is always satisfied but for the Taylor rule model this is not the case due to implicit prior truncation as a result of the Blanchard-Khan conditions not being satisfied. To alleviate this, when we perform model comparison we use a uniform prior for ϕ_π in the range between 1 and 3, meaning the Blanchard-Khan conditions are satisfied at all parts of the prior mass and there is no truncation. Posterior estimates are practically indistinguishable from the ones in Table A.1 and so we do not report them. Let $\log\left(p\left(Z_{1:T} \mid \mathcal{M}_j\right)\right)$ denote the log marginal likelihood of model j for data vector $Z_{1:T}$. For the Taylor rule model, we obtain $\log\left(p\left(Z_{1:T} \mid \mathcal{M}_{TR}\right)\right) = -489.1$, whereas for the state rule model, we obtain $\log\left(p\left(Z_{1:T} \mid \mathcal{M}_{TR}\right)\right) = -459.3$. Therefore the data overwhelmingly prefers the state rule model. To ascertain the degree to which the state rule model is preferred, we compute odds ratios and estimate a posterior density over the pair of models. We use non-informative priors, i.e. $p(\mathcal{M}_{TR}) = p(\mathcal{M}_{RR}) = 0.5$. Using these priors, we obtain a posterior probability on the state rule model of 1 and the state rule model is favoured decisively.

2.3 Pre- and Post-Volcker

Lubik and Schorfheide [2004] present evidence that prior to Paul Volcker becoming chairman of the Federal Reserve, US monetary policy was inconsistent with determinacy. Through the lens of a very similar stylised New Keynesian model which features a Taylor rule, by deviating from typical estimation methods and allowing for indeterminacy they find that indeterminacy was indeed present prior to 1980. To a similar end, we re-estimate theh model with a state rule in the pre- and post-Volcker periods. The pre-Volcker period is defined as `1959Q1-1979Q2`, while the post-Volcker period is defined as `1982Q4-2019Q4`, meaning that the Volcker disinflation period is excluded. Table 4 presents the implied Taylor rules and value of the determinacy condition (if monetary policy was specified as following the implied Taylor rule) for each period, evaluated at the respective posterior mean.[9]

A key result is that the determinacy condition (21) is not satisfied in the pre-Volcker period, but is satisfied in the post-Volcker period, aligning with the findings in Lubik and Schorfheide [2004]. $\Omega > 1$ is the condition for determinacy of the equilibrium under a Taylor rule. At the posterior mean, we have $\Omega = 0.72$ in the pre-Volcker period and $\Omega = 1.38$ in the post-Volcker period. Figure 5 plots the posterior distribution of the determinacy condition in the two periods, clearly illustrating that in the pre-Volcker period the majority of the posterior mass is in the $\Omega < 1$ region, whereas in the post-Volcker period the majority of the posterior mass is in the $\Omega > 1$ region. We have $Pr(\Omega \leq 1) = 0.71$ in the pre-Volcker period and $Pr(\Omega \leq 1) = 0.13$ in the post-Volcker period, indicative of a shift to the determinacy region.

A second striking result comes with respect to the stability of the Phillips curve over time. If one restimates[10] the model over the two subsamples with a Taylor rule model (ruling out by assumption indeterminacy), then there appears to be a significant structural break between the two periods for the slope of the Phillips curve, as it becomes much steeper in the Post-Volcker era (from $\kappa = 0.220$ pre-Volcker to $\kappa = 0.745$ post-Volcker with respective 95% interval [0.11 0.33] and [0.69 0.79]). In the state rule model, however, the slope does not change between the two periods and remains flat in both ($\kappa = 0.006$ pre-Volcker and $\kappa = 0.005$ post-Volcker). This again implies that the restriction to the determinacy region in the estimation of the Taylor rule model is influencing the estimate of structural parameters. This result aligns with Hazell, Herreño, Nakamura, and Steinsson [2022], who find, using state-level data on inflation and unemployment, that the Phillips curve has consistently been flat over time.

Furthermore, the state rule model provides a better fit to the data in both periods. In the pre-Volcker period, we obtain $\log\left(p\left(Z_{1:T_{Pre}} \mid \mathcal{M}_{TR}\right)\right) = -181.5$, whereas for the

[9] Tables A.3 and A.4 in the Appendix present the posterior estimates for both models in each period.
[10] See Tables A.5 and A.6 in the Appendix

Table 4: Implied Taylor Rule and Determinacy Condition, Pre-Volcker and Post-Volcker

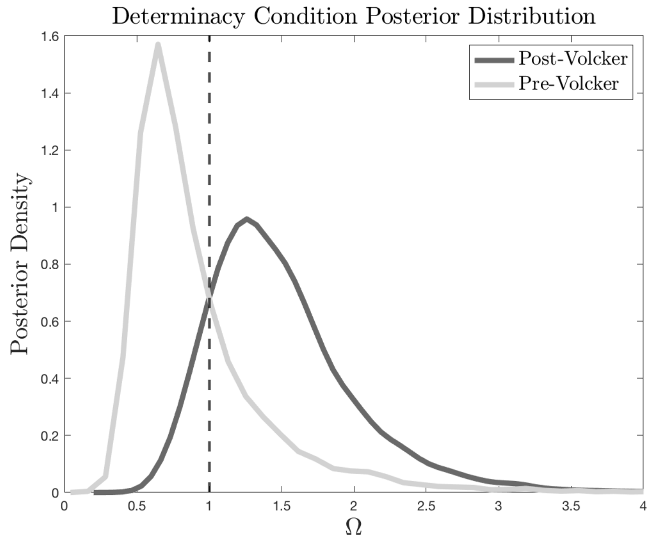
	Pre-Volcker		Post-Volcker	
	Taylor R.	State R. Implied	Taylor R.	State R. Implied
ϕ_π	1.79	0.24	2.19	-0.16
ϕ_y	0.06	0.29	0.01	0.77
Ω	0.72		1.38	

Notes: The estimated model is the simple three-equation New Keynesian model with a Taylor rule or state rule. The Taylor rule columns present the posterior mean estimates of the Taylor rule parameters, while the state rule columns present the implied Taylor rule obtained from the model with all parameters at the posterior mean. The state rule model is also used to obtain the posterior mean for Ω which is the determinacy condition. The condition for determinacy is $\Omega > 1$. This is performed for estimations from the pre-Volcker and post-Volcker periods. Samples are `1959Q1-1979Q2` *and* `1982Q4-2019Q4` *respectively.*

state rule model, we obtain $\log\left(p\left(Z_{1:T_{Pre}} \mid \mathcal{M}_{RR}\right)\right) = -172.6$. In the post-Volcker period, we obtain $\log\left(p\left(Z_{1:T_{Post}} \mid \mathcal{M}_{TR}\right)\right) = -217.9$, whereas for the state rule model, we obtain $\log\left(p\left(Z_{1:T_{Post}} \mid \mathcal{M}_{RR}\right)\right) = -152.4$. This results in a posterior model probability extremely close to 1 in both cases for the state rule model.

There is clear evidence from the state rule model suggesting that monetary policy did change post-Volcker, becoming more hawkish, and this was how determinacy was achieved. Specifically, the estimates suggest that the Federal reserve responded much more aggressively to 'demand' shocks, raising the real interest rate by almost twice as much after Volcker than it did before. We also estimate that it lowered real interest rates by a lesser degree in response to supply shocks post-Volcker, by around half as much as previously. In the Taylor rule model, the inflation coefficient increases, as is typically associated with more hawkish monetary policy post-Volcker. In the pre-Volcker period, the coefficient is still estimated to be well above one however, in contrast to Clarida, Galí, and Gertler [2000], who present reduced form evidence that the coefficient was below one before during this period, . This is also what Lubik and Schorfheide [2004] find when they estimate a DSGE model and allow for indeterminacy. The implied Taylor rule from the state rule model estimates actually suggest that the coefficient on inflation fell between the two monetary regimes. As the prior passage outlined, however, this is still consistent with a more hawkish shift in monetary policy. The two models draw differing conclusions on the changing role of discretionary monetary policy (the monetary shock). The Taylor rule model estimates show that the standard deviation of monetary shocks increased post-Volcker, whereas the state rule model finds a small decrease. Both models agree that the volatility of demand shocks

Figure 5: Posterior Distribution of the Determinacy Condition



Notes: The dotted black line represents the boundary between the indeterminacy and determinacy regions. The condition for determinacy is $\Omega > 1$. Estimated model is the simple three-equation New Keynesian model with a state rule. Samples are `1959Q1-1979Q2` *and* `1982Q4-2019Q4`*.*

decreased post-Volcker and that the volatility of supply shocks increased.

2.4 An Extended Three-Equation Model

The fully forward three equation New Keynesian model is clearly stylised, and so we estimate an extended version of the model which includes habit persistence and a hybrid Phillips curve

$$\begin{aligned} y_t &= \alpha_y \left((1-\alpha_1)E_t y_{t+1} + \alpha_1 y_{t-1}\right) - \frac{1}{\gamma}\left(i_t - E_t \pi_{t+1}\right) + d_t, \\ \pi_t &= \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} + \kappa y_t + \nu_t, \end{aligned}$$

where:

$$\begin{aligned} \alpha_1 &= \frac{\lambda}{1+\lambda}, \\ \pi_1 &= \frac{\beta}{1+\beta\iota}, \\ \pi_2 &= \frac{\iota}{1+\beta\iota}, \end{aligned} \tag{22}$$

and with λ denoting the habits parameter in the utility function and ι denoting the degree of price indexation to past inflation. This collapses to the standard three equation model when $\iota = \lambda = 0$. Again, we assume quasi-no Euler discounting by setting $\alpha_y = .999$. The model is closed by a standard Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + \nu_t,$$

or a state real interest rate rule:

$$i_t - E_t\pi_{t+1} = \sigma_{y_{-1}} y_{t-1} + \sigma_{\pi_{-1}} \pi_{t-1} + \sigma_d d_t + \sigma_\mu \mu_t + \nu_t$$

Table A.7 in the appendix presents prior and posterior distributions from the Taylor rule model, while Table A.8 presents estimates from the state rule model. The two key results we have obtained with the fully forward model remain present: *i)* the estimate of the Phillips curve slope is much lower in the state rule model (0.004) than in the Taylor rule model (0.682) *ii)* the data overwhelmingly prefers the state rule model, with $\log(p(Z_{1:T} \mid \mathcal{M}_{NK+,TR})) = -483.5$, and $\log(p(Z_{1:T} \mid \mathcal{M}_{NK+,RR})) = -458.7$ for a posterior model probability of 1 for the latter. Additionally, the state rule model estimates a much larger value of λ, the habits parameter as well as ι, the price indexation parameter. The implied Taylor rule at the posterior mean of the state rule model yields $\phi_\pi = -0.20$ and $\phi_y = 0.65$, which would create indeterminacy if implemented in the model.

The Taylor rule we have used in the estimation maps a linear space of dimension two (plus the monetary shock) to the instrument, while the state rule uses a linear space of dimension four (plus the monetary shock). The bias could come from determinacy or from misspecification (or from both). To sort this out, we reestimate the model with an augmented Taylor rule that uses the same linear space than the state rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + \phi_{y_{-1}} y_{t-1} + \phi_{\pi_{-1}} \pi_{t-1} + \nu_t,$$

Estimation results are presented in Table A.9. Note that parameters estimates are still very different from the ones obtained with the state rule. In particular, the slope of the Phillips curve parameter κ posterior mean is large (0.483). As there is no misspecification in that estimation, the bias has to be a determinacy bias. Indeed, if we use the implied Taylor rule, as obtained from the estimation of the state rule model, in a model with coefficient estimated with the augmented Taylor rule, we find that Blanchard and Kahn's [1980] condition is not satisfied and that we have indeterminacy.

2.5 A Heterogeneous-Agent New Keynesian Model

We further extend the model and now estimate the sticky wage Heterogeneous-Agent New Keynesian (HANK) model of Broer, Hansen, Krusell, and Öberg [2020]. The model features workers and capitalists, with a no-borrowing constraint and zero net supply of bonds resulting in a degenerate wealth distribution in equilibrium. Wage setting is subject to Rotemberg's [1982] adjustment costs, leading to a wage Phillips curve in addition to the standard price Phillips curve. Once again we introduce demand, supply and monetary shocks into the framework, and also add a wage setting shock and quasi-no Euler discounting ($\alpha_y = .999$). We refer to Broer, Hansen, Krusell, and

Öberg [2020] for the derivation of the set of equilibrium conditions in the linearised model, which are given by

$$y_t = \alpha_y E_t y_{t+1} - \frac{1}{\gamma}\left(i_t - E_t \pi^p_{t+1}\right) + d_t, \quad (23)$$

$$\pi^p_t = \beta E_t \pi^p_{t+1} + \kappa w_t + \mu_t, \quad (24)$$

$$\pi^w_t = \beta E_t \pi^w_{t+1} - \kappa_w \left(w_t - (y_t + \psi n_t)\right) + \mu^w_t, \quad (25)$$

$$w_t = w_{t-1} + \pi^w_t - \pi^p_t, \quad (26)$$

$$y_t = w_t + n_t. \quad (27)$$

Equation (23) is the Euler equation, (24) the Phillips curve, (25) the wage Philips curve, (26) the wage accounting euqation and (27) is market clearing condition.

The model is closed by either the Taylor rule the authors have specified:

$$i_t = \phi_\pi \pi^p_t + \nu_t,$$

or the real interest rate state rule:

$$i_t - E_t \pi_{t+1} = \sigma_w w_{t-1} + \sigma_d d_t + \sigma_\mu \mu_t + \sigma_{\mu^w} \mu^w_t + \nu_t$$

We again estimate the two models over the period 1959:I - 2019:IV. The data we use are the Fed Funds rate, CPI inflation rate, the growth rate of real compensation per hour and the growth rate of real consumption. We calibrate $\beta = 0.99$, $\gamma = 1$, $\psi = 1$. Table A.10 displays the prior and posterior distributions for the estimated parameters in the model with a Taylor rule, while Table A.11 displays these for the model with a state rule.

Once again, the most notable result is that the state rule model estimates a much flatter Phillips curve than the Taylor rule model by a substantial degree. This is also true for the slope of the wage Phillips curve, although the differential is smaller. The state rule parameter estimates imply that the Federal reserve raises the real interest rate in response to demand shocks, and lowers it in response to supply and wage shocks. A model comparison reveals that the state rule model is again overwhelmingly preferred. We obtain $\log\left(p\left(Z_{1:T} \mid \mathcal{M}_{HANK,TR}\right)\right) = -796.8$, whereas for the state rule model, we obtain $\log\left(p\left(Z_{1:T} \mid \mathcal{M}_{HANK,RRR}\right)\right) = -718.1$. With uninformative priors, this gives a posterior model probability of 1 for the state rule model. Generating simulated data at with parameters at the posterior mean and estimating an implied Taylor rule in the state rule model, we obtain $\phi_\pi = -0.28$. We verify that this parameter configuration does indeed imply indeterminacy in the Taylor rule model, suggesting that this is a significant issue for the estimates in this HANK model.

3 The Smets and Wouters's [2007] Medium Scale Model

We have seen in the previous section that for relatively small scale models, determinacy bias is pervasive. We now consider a medium scale model that has many dimensions of backwardness and that should be less prone to indeterminacy, but has larger state space, which should makes misspecification bias more likely. The model is the canonical medium scale New Keynesian model of Smets and Wouters [2007].

3.1 Estimation

The sample period is extended to be `1959Q1-2019Q4`, and the same observable variables are used as in Smets and Wouters [2007] with the only difference being the use of the shadow Federal Funds rate of Wu and Xia [2016] from `2007Q1` onwards when the zero lower bound was binding.

The model equations are the same than in Smets and Wouters [2007] except for the introduction of a quasi-no discounted Euler equation. We first estimate the model under an extended specification of the Taylor rule, as assumed by Smets and Wouters [2007]:

$$r_t = \rho r_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y(y_t - y_t^p)) + \phi_{\Delta y}((y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)) + \varepsilon_t^R$$

where y_t^f is the flex price level of output and $y_t - y_t^f$ is the output gap. To maintain the same notation as Smets and Wouters [2007], we now let r_t denote the nominal interest rate while i_t is investment. As previously, we also perform the model estimation under the real interest rate state rule

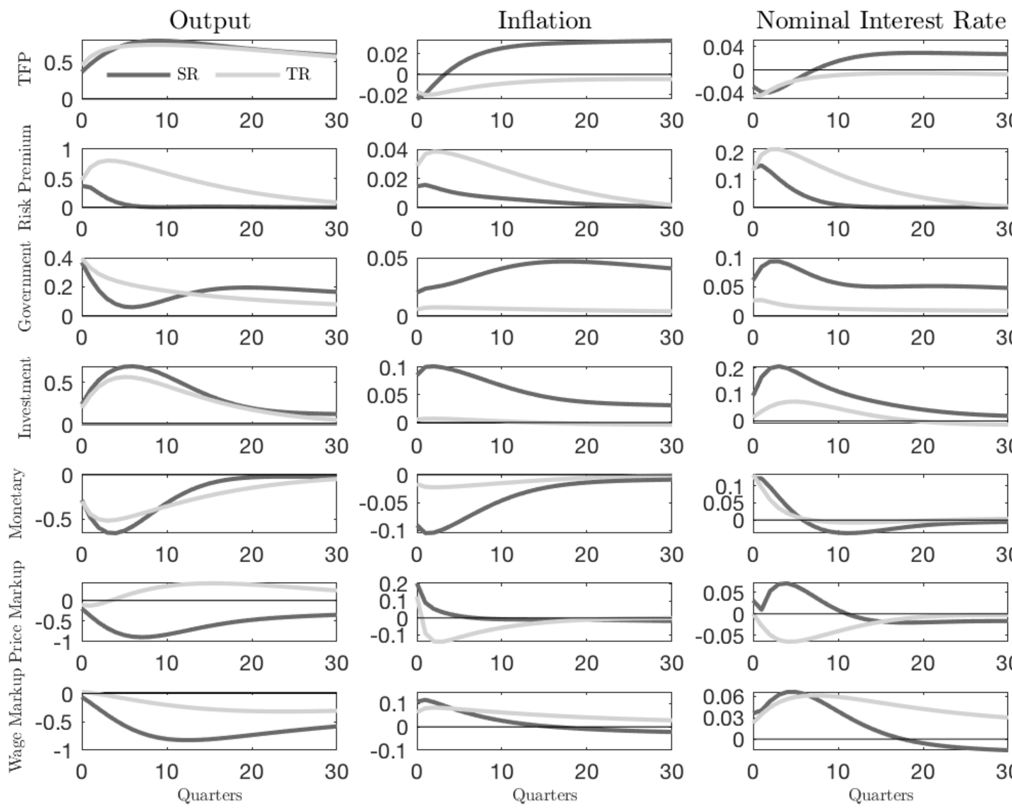
$$\begin{aligned} r_t - E_t\pi_{t+1} &= \psi_a \varepsilon_t^a + \psi_b \varepsilon_t^b + \psi_g \varepsilon_t^g + \psi_{i^s}\varepsilon_t^i + \psi_{p^{inf}}\varepsilon_t^p + \psi_{w^s}\varepsilon_t^w + \psi_{\eta^w}\eta_{t-1}^w \\ &+ \psi_{\eta^p}\eta_{t-1}^p + \psi_{y^p} y_{t-1}^p + \psi_y y_{t-1} + \psi_r r_{t-1} + \psi_{k^{p,s}} k_t^{p,s} + \psi_{k^s} k_t^s \\ &+ \psi_{c^p} c_{t-1}^p + \psi_{i^p} i_{t-1}^p + \psi_c c_{t-1} + \psi_i i_{t-1} + \psi_\pi \pi_{t-1} + \psi_w w_{t-1} + \varepsilon_t^R. \end{aligned}$$

Tables A.12 and A.13 present results from estimating the original Smets and Wouters's [2007] model over the extended sample, and tables A.14 and A.15 show results from estimation under the state rule. There are so many parameters that discussing differences between estimates one by one is both fastidious and difficult to interpret. We rather look at some moments of the two models. We first compute the slope of the Phillips curve at the posterior mean, which is given by

$$\kappa = 1/\left(1 + \beta\gamma^{1-\sigma_c}\iota_p\right)\left[\left(1 - \ \beta\gamma^{1-\sigma_c}\xi_p\right)\left(1-\xi_p\right)/\xi_p\left(\left(\phi_p - 1\right)\varepsilon_p + 1\right)\right]$$

The estimated value of κ is .003 with the Taylor rule and .019 with the state rule. The difference between these two estimates is not significantly different from zero at conventional levels. To get a feel of how different are the two estimations, we plot on Figure 6 the impulse responses of the output gap, inflation and the real interest rate to the seven shocks in the model, when parametrs are at the respective posterior mean of the two versions of the model. We see clearly that responses are very different.

Figure 6: Responses to Shocks in the Smets and Wouters's [2007] Model with a Taylor Rule or a State Rule



Notes: The light grey line corresponds to the Taylor rule version of the Smets and Wouters (2007) Model and the dark grey line to the state rule version. In each case, all parameters are set to their respective posterior mean values from the model's estimation. Sample is 1959Q1-2019Q4.

Again, instead of commenting each and every response, let us focus on inflation, which is a key variable for the users of such models, especially Central Banks. There are sharp differences in the responses of inflation to virtually all the shocks, except perhaps the wage markup shock. To better grasp how different is the model depending on the specification of the policy rule, Table 5 reports the results of an unconditional variance decomposition of inflation for the state rule and Taylor rule models. Table A.16 in the appendix repeats this for the output gap.

Table 5: Unconditional Variance Decomposition of Inflation

	State rule	Taylor Rule
TFP	16	2
Risk Premium	0	6
Government Spending	16	1
Investment Technology	21	0
Monetary Policy	13	2
Price Markup	14	47
Wage Markup	19	41

Notes: The estimated models are the Smets and Wouters's [2007] model with either a Taylor rule or a state rule version. Sample is `1959Q1-2019Q4`*. The sum of the shares may not add up to one because of the rounding.*

The model with a Taylor rule ascribes almost all of inflation's unconditional variance (88%) to the two markup shocks. This is a challenge for the practical use of the model for policy advising, as these shocks are most likely catch-all shocks with little observed empirical counterparts. The model with a state rule model gives a totally different account of inflation fluctuations, as estimates show that six of the seven shocks contribute to roughly one-sixth of the unconditional variance of inflation.

3.2 The Missepecification Bias

It is important to remind the reader that the state rule specification encompasses the Taylor rule one. In other words, the Taylor rule is putting constraints on the way monetary authorities respond to the state of the economy. If these constraints were present in the Data Generating Process, then the two estimations would give the same results. One could be doubtful that monetary authorities can observe and react freely to the twenty state variables of the Smets and Wouters's [2007] model. But we are not imposing it with our state rule, as some coefficients could be estimated to zero if the actual information set of the Fed was coarser.

The differences between the parameters estimates with the two rules (state and Taylor) indicate that we have a bias. We first check if it is a determinacy bias by computing the implied Taylor rule of the model estimated with a state rule. We generate 1,000,000 periods of simulated data, without the monetary shock (which is equivalent to estimating the Taylor rule instrumenting for all the models shocks but the monetary policy one). Table 6 reports the estimates for the implied Taylor rule in the state rule model alongside the Taylor rule parameter estimates for the model which features this explicitly as the monetary policy rule.

Table 6: Estimated State Rule and Implied Implied Taylor Rules

	Implied Taylor R.	Estimated Taylor R.
ϕ_π	1.43	1.84
ϕ_y	0.00	0.11
ρ	0.84	0.87
$\phi_{\Delta y}$	0.17	0.25

Notes: The estimated model is the Smets and Wouters's [2007] model with a state rule version. The left column displays the estimates Taylor rule parameters obtained from simulated data from this model. The right column displays the posterior mean of the Taylor rule parameters in the model which explicitly features a Taylor rule. Sample is `1959Q1-2019Q4`.

Interestingly, the R^2 for the implied Taylor rule is 0.97, despite the estimated coefficients differing substantially from the model where it is featured explicitly. As such, if an econometrician was estimating a Taylor rule on the generated data, they would conclude that it has a very good fit, even though in reality this is not the rule that the monetary authorities are following. A common justification for the use of Taylor rules in New Keynesian models is that they provide a good fit to the data, but this exercise illustrates that this can occur even if the underlying rule takes a very different form. We see that the implied Taylor rule will satisfy the Taylor principle, so that no determinacy bias is at play here. The discrepancy between the (constrained) Taylor rule estimation and the (unconstrained) state rule one is therfore explained by a misspecification bias. In other words, Smets and Wouters's [2007] specification of the Taylor rule puts constraints on monetary policy that are rejected by the data, and that bias the estimation of the model deep parameters.

4 Conclusion

Our analysis started from two observations. First, the specification of how monetary policy is conducted matters for equilibrium properties of New Keynesian models. Second, the applied literature that estimates New Keynesian models does not justify the specific specification of the Taylor rule that is assumed, as if it should not really matter as long as it broadly aligns with the Taylor narrative ("tighten when inflation is high, loosen when output gap is high"). We show that this practice is actually harmful, as it is subject to a determinacy and a misspecification bias. We use simple abstract models to theoretically uncover these biases, and then show that they indeed materialise in small and medium size New Keynesian models, when estimated on US data. The practical conclusion we draw from our work is the following advice to practitioners.

Models estimated with an incarnation of the Taylor rule should be re-estimated with the state rule we have proposed in this paper. If there is a significant difference in the deep parameters estimates between the two estimations, then the state rule estimation should be preferred.

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Appendix

A Estimation Results

Table A.1: Simple New Keynesian Model with a Taylor Rule Model

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.678	0.0626	0.5869	0.7849
ϕ_π	norm	1.500	0.2500	1.769	0.1741	1.4906	2.0523
ϕ_y	norm	0.125	0.0250	-0.014	0.0128	-0.0340	0.0035
ρ_d	beta	0.500	0.2000	0.962	0.0121	0.9417	0.9813
ρ_μ	beta	0.500	0.2000	0.994	0.0031	0.9895	0.9989
ρ_ν	beta	0.500	0.2000	0.457	0.0182	0.4275	0.4873
s_d	invg	0.100	2.0000	0.092	0.0129	0.0707	0.1128
s_μ	invg	0.100	2.0000	0.575	0.0535	0.4850	0.6591
s_ν	invg	0.100	2.0000	0.811	0.0878	0.6682	0.9469

Notes: Estimated model is the simple three-equation New Keynesian model with a Taylor rule. Sample is 1959Q1-2019Q4.

Table A.2: Simple New Keynesian Model with a State Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.006	0.0014	0.0036	0.0080
σ_d	unif	0.000	1.1547	0.971	0.0145	0.9493	0.9934
σ_μ	unif	0.000	1.1547	-0.463	0.0657	-0.5691	-0.3562
ρ_d	beta	0.500	0.2000	0.992	0.0038	0.9859	0.9977
ρ_μ	beta	0.500	0.2000	0.371	0.0317	0.3187	0.4218
ρ_ν	beta	0.500	0.2000	0.924	0.0104	0.9069	0.9412
s_d	invg	0.100	2.0000	0.102	0.0134	0.0793	0.1232
s_μ	invg	0.100	2.0000	0.324	0.0235	0.2858	0.3630
s_ν	invg	0.100	2.0000	0.057	0.0077	0.0446	0.0699

Notes: Estimated model is the simple three-equation New Keynesian model with a state rule. Sample is 1959Q1-2019Q4.

Table A.3: Simple New Keynesian Model with a State Rule, Pre-Volcker

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.006	0.0033	0.0012	0.0109
σ_d	unif	0.000	1.1547	0.518	0.0898	0.3765	0.6663
σ_μ	unif	0.000	1.1547	-0.863	0.2461	-1.2662	-0.4582
ρ_d	beta	0.500	0.2000	0.739	0.0788	0.6141	0.8654
ρ_μ	beta	0.500	0.2000	0.727	0.0431	0.6577	0.7980
ρ_ν	beta	0.500	0.2000	0.863	0.0842	0.7942	0.9519
s_d	invg	0.100	2.0000	0.372	0.0702	0.2587	0.4846
s_μ	invg	0.100	2.0000	0.112	0.0233	0.0745	0.1484
s_ν	invg	0.100	2.0000	0.098	0.0280	0.0523	0.1385

Notes: Estimated model is the simple three-equation New Keynesian model with a state rule. Sample is 1959Q1-1979Q2.

Table A.4: Simple New Keynesian Model with a State Rule, Post-Volcker

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.005	0.0015	0.0028	0.0077
σ_d	unif	0.000	1.1547	0.969	0.0156	0.9452	0.9929
σ_μ	unif	0.000	1.1547	-0.215	0.0604	-0.3126	-0.1146
ρ_d	beta	0.500	0.2000	0.982	0.0064	0.9725	0.9928
ρ_μ	beta	0.500	0.2000	0.179	0.0442	0.1070	0.2527
ρ_ν	beta	0.500	0.2000	0.938	0.0096	0.9223	0.9537
s_d	invg	0.100	2.0000	0.095	0.0098	0.0790	0.1110
s_μ	invg	0.100	2.0000	0.367	0.0290	0.3195	0.4139
s_ν	invg	0.100	2.0000	0.035	0.0055	0.0255	0.0435

Notes: Estimated model is the simple three-equation New Keynesian model with a state rule. Sample is 1982Q4-2019Q4.

Table A.5: Simple New Keynesian Model with a Taylor Rule, Pre-Volcker

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.220	0.0661	0.1135	0.3291
ϕ_π	norm	1.500	0.2500	1.789	0.2871	1.2577	2.2433
ϕ_y	norm	0.125	0.0250	0.061	0.0389	-0.0088	0.1174
ρ_d	beta	0.500	0.2000	0.839	0.0921	0.7248	0.9648
ρ_μ	beta	0.500	0.2000	0.989	0.0066	0.9799	0.9984
ρ_ν	beta	0.500	0.2000	0.634	0.0656	0.5192	0.7322
s_d	invg	0.100	2.0000	0.157	0.0903	0.0325	0.2338
s_μ	invg	0.100	2.0000	0.240	0.0775	0.1208	0.3761
s_ν	invg	0.100	2.0000	0.612	0.1157	0.4098	0.8044

Notes: Estimated model is the simple three-equation New Keynesian model with a Taylor rule. Sample is 1959Q1-1979Q2.

Table A.6: Simple New Keynesian Model with a Taylor Rule, Post-Volcker

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.745	0.0387	0.6898	0.7925
ϕ_π	norm	1.500	0.2500	2.190	0.1983	1.8639	2.5175
ϕ_y	norm	0.125	0.0250	0.011	0.0213	-0.0242	0.0452
ρ_d	beta	0.500	0.2000	0.976	0.0096	0.9607	0.9916
ρ_μ	beta	0.500	0.2000	0.993	0.0036	0.9881	0.9987
ρ_ν	beta	0.500	0.2000	0.423	0.0127	0.4025	0.4437
s_d	invg	0.100	2.0000	0.071	0.0080	0.0573	0.0838
s_μ	invg	0.100	2.0000	0.480	0.0338	0.4248	0.5360
s_ν	invg	0.100	2.0000	1.002	0.0996	0.8409	1.1661

Notes: Estimated model is the simple three-equation New Keynesian model with a Taylor rule. Sample is 1982Q4-2019Q4.

Table A.7: Extended New Keynesian Model Taylor Rule Model

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.682	0.0632	0.5961	0.7924
ϕ_π	norm	1.500	0.2500	1.594	0.1279	1.3842	1.7995
ϕ_y	norm	0.125	0.0500	-0.064	0.0147	-0.0870	-0.0394
ι	beta	0.500	0.2000	0.031	0.0173	0.0047	0.0566
λ	beta	0.500	0.2000	0.104	0.0345	0.0477	0.1600
ρ_d	beta	0.500	0.2000	0.965	0.0117	0.9464	0.9846
ρ_μ	beta	0.500	0.2000	0.989	0.0057	0.9804	0.9977
ρ_ν	beta	0.500	0.2000	0.445	0.0192	0.4144	0.4775
s_d	invg	0.100	2.0000	0.077	0.0108	0.0587	0.0943
s_μ	invg	0.100	2.0000	0.616	0.0543	0.5269	0.7054
s_ν	invg	0.100	2.0000	0.738	0.0629	0.6373	0.8409

Notes: Estimated model is the extended three-equation New Keynesian model with a Taylor rule. Sample is 1959Q1-2019Q4.

Table A.8: Extended New Keynesian Model with State Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.004	0.0011	0.0020	0.0056
σ_d	unif	0.000	1.1547	0.741	0.0906	0.6039	0.8833
σ_μ	unif	0.000	1.1547	-0.389	0.0677	-0.4991	-0.2769
$\sigma_{y_{-1}}$	unif	0.000	1.1547	0.059	0.0128	0.0384	0.0803
$\sigma_{\pi_{-1}}$	unif	0.000	1.1547	-0.033	0.0294	-0.0801	0.0160
ι	beta	0.500	0.2000	0.202	0.0686	0.0908	0.3150
λ	beta	0.500	0.2000	0.291	0.0741	0.1674	0.4130
ρ_d	beta	0.500	0.2000	0.991	0.0044	0.9840	0.9976
ρ_μ	beta	0.500	0.2000	0.148	0.0644	0.0445	0.2529
ρ_ν	beta	0.500	0.2000	0.929	0.0129	0.9075	0.9492
s_d	invg	0.100	2.0000	0.096	0.0150	0.0722	0.1206
s_μ	invg	0.100	2.0000	0.369	0.0255	0.3270	0.4104
s_ν	invg	0.100	2.0000	0.090	0.0105	0.0720	0.1062

Notes: Estimated model is the extended three-equation New Keynesian model with a state rule. Sample is 1959Q1-2019Q4.

Table A.9: Extended New Keynesian Model with Extended Taylor Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.483	0.0708	0.3662	0.5959
ϕ_π	norm	1.500	0.2500	1.772	0.1387	1.5442	1.9984
ϕ_y	norm	0.125	0.0500	0.233	0.0425	0.1644	0.3041
$\phi_{\pi_{-1}}$	norm	0.000	0.2500	-0.086	0.0834	-0.2198	0.0509
$\phi_{y_{-1}}$	norm	0.000	0.2500	-0.342	0.0464	-0.4173	-0.2651
ι	beta	0.500	0.2000	0.049	0.0268	0.0075	0.0888
λ	beta	0.500	0.2000	0.287	0.0459	0.2105	0.3611
ρ_d	beta	0.500	0.2000	0.968	0.0110	0.9496	0.9857
ρ_μ	beta	0.500	0.2000	0.954	0.0181	0.9250	0.9835
ρ_ν	beta	0.500	0.2000	0.331	0.0366	0.2714	0.3908
s_d	invg	0.100	2.0000	0.080	0.0098	0.0638	0.0960
s_μ	invg	0.100	2.0000	0.514	0.0533	0.4250	0.5975
s_ν	invg	0.100	2.0000	0.877	0.0734	0.7564	0.9947

Notes: Estimated model is the extended three-equation New Keynesian model with a longer Taylor rule. Sample is `1959Q1-2019Q4`.

Table A.10: Broer, Hansen, Krusell, and Öberg's [2020] HANK Model with a Taylor Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.723	0.0494	0.6541	0.7925
κ_w	gamm	0.100	0.0500	0.154	0.0437	0.0843	0.2201
ϕ_π	norm	1.500	0.2500	1.792	0.1397	1.5618	2.0162
ρ_d	beta	0.500	0.2000	0.964	0.0119	0.9442	0.9832
ρ_μ	beta	0.500	0.2000	0.988	0.0057	0.9792	0.9970
ρ_ν	beta	0.500	0.2000	0.445	0.0164	0.4194	0.4725
ρ_w	beta	0.500	0.2000	0.991	0.0045	0.9848	0.9986
s_d	invg	0.100	2.0000	0.096	0.0099	0.0790	0.1114
s_μ	invg	0.100	2.0000	0.589	0.0425	0.5183	0.6575
s_ν	invg	0.100	2.0000	0.823	0.0728	0.7043	0.9414
s_w	invg	0.100	2.0000	0.275	0.0783	0.1488	0.3924

Notes: Estimated model is the HANK model with a Taylor rule. Sample is `1959Q1-2019Q4`.

Table A.11: Broer, Hansen, Krusell, and Öberg's [2020] HANK Model with a State Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
κ	gamm	0.100	0.0500	0.003	0.0008	0.0015	0.0041
κ_w	gamm	0.100	0.0500	0.035	0.0153	0.0151	0.0553
σ_d	unif	0.000	1.1547	0.936	0.0250	0.8955	0.9766
σ_μ	unif	0.000	1.1547	-0.438	0.0710	-0.5517	-0.3179
σ_{μ^w}	unif	0.000	1.1547	-0.467	0.2640	-0.8523	-0.0785
σ_w	unif	0.000	1.1547	0.057	0.0133	0.0355	0.0790
ρ_d	beta	0.500	0.2000	0.968	0.0217	0.9393	0.9995
ρ_μ	beta	0.500	0.2000	0.403	0.0364	0.3450	0.4639
ρ_ν	beta	0.500	0.2000	0.965	0.0085	0.9516	0.9794
ρ_w	beta	0.500	0.2000	0.979	0.0083	0.9658	0.9921
s_d	invg	0.100	2.0000	0.203	0.0215	0.1641	0.2345
s_μ	invg	0.100	2.0000	0.314	0.0249	0.2723	0.3536
s_i	invg	0.100	2.0000	0.051	0.0092	0.0361	0.0662
s_w	invg	0.100	2.0000	0.067	0.0265	0.0319	0.1015

Notes: Estimated model is the HANK model with a state rule. Sample is 1959Q1-2019Q4.

Table A.12: Parameter Estimates from Smets and Wouters [2007] with a Taylor Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ_a	beta	0.500	0.2000	0.985	0.0059	0.9762	0.9945
ρ_b	beta	0.500	0.2000	0.884	0.0309	0.8345	0.9338
ρ_g	beta	0.500	0.2000	0.975	0.0072	0.9635	0.9870
ρ_i	beta	0.500	0.2000	0.813	0.0557	0.7218	0.9039
ρ_r	beta	0.500	0.2000	0.134	0.0507	0.0483	0.2142
ρ_p	beta	0.500	0.2000	0.919	0.0232	0.8837	0.9565
ρ_w	beta	0.500	0.2000	0.973	0.0156	0.9537	0.9936
μ_p	beta	0.500	0.2000	0.979	0.0103	0.9636	0.9955
μ_w	beta	0.500	0.2000	0.952	0.0203	0.9257	0.9809
φ	norm	4.000	1.5000	4.184	0.9269	2.6273	5.6402
σ_c	norm	1.500	0.3750	1.428	0.1649	1.1586	1.6952
λ	beta	0.700	0.1000	0.496	0.0485	0.4173	0.5769
ξ_w	beta	0.500	0.1000	0.819	0.0355	0.7630	0.8778
σ_l	norm	2.000	0.7500	1.765	0.4890	0.9710	2.5628
ξ_p	beta	0.500	0.1000	0.862	0.0239	0.8223	0.9005
ι_w	beta	0.500	0.1500	0.500	0.1241	0.2956	0.7050
ι_p	beta	0.500	0.1500	0.296	0.0820	0.1603	0.4294
ψ	beta	0.500	0.1500	0.692	0.0918	0.5411	0.8420
ϕ_p	norm	1.250	0.1250	1.491	0.0737	1.3693	1.6112
r_π	norm	1.500	0.2500	1.842	0.1628	1.5751	2.1103
ρ	beta	0.750	0.1000	0.870	0.0180	0.8410	0.9000
r_y	norm	0.125	0.0500	0.113	0.0237	0.0744	0.1519
$r_{\Delta y}$	norm	0.125	0.0500	0.254	0.0241	0.2140	0.2933
$\bar{\pi}$	gamm	0.625	0.1000	0.789	0.1042	0.6150	0.9571
$100(\beta^{-1}-1)$	gamm	0.250	0.1000	0.132	0.0501	0.0515	0.2093
$\bar{l}$	norm	0.000	2.0000	1.433	1.1215	-0.4214	3.2585
$\bar{\gamma}$	norm	0.400	0.1000	0.390	0.0255	0.3487	0.4308
ρ_{ga}	norm	0.500	0.2500	0.565	0.0632	0.4622	0.6696
α	norm	0.300	0.0500	0.186	0.0163	0.1591	0.2128

Notes: Estimated model is the Smets and Wouters's [2007] model with a Taylor rate. Sample is 1959Q1-2019Q4.

Table A.13: Parameter Estimates from Smets and Wouters [2007] with a Taylor Rule (Standard Deviation of Structural Shocks), Sample:1959:I-2019:IV

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
η^a	invg	0.100	2.0000	0.491	0.0267	0.4471	0.5345
η^b	invg	0.100	2.0000	0.087	0.0099	0.0713	0.1036
η^g	invg	0.100	2.0000	0.465	0.0224	0.4287	0.5018
η^i	invg	0.100	2.0000	0.345	0.0339	0.2890	0.3986
η^m	invg	0.100	2.0000	0.217	0.0120	0.1977	0.2368
η^p	invg	0.100	2.0000	0.194	0.0141	0.1710	0.2169
η^w	invg	0.100	2.0000	0.334	0.0188	0.3026	0.3642

Notes: Estimated model is the Smets and Wouters's [2007] model with a Taylor rate. Sample is 1959Q1-2019Q4.

Table A.14: Parameter Estimates from Smets and Wouters [2007] with a State Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ_a	beta	0.500	0.2000	0.982	0.0069	0.9715	0.9934
ρ_b	beta	0.500	0.2000	0.275	0.0805	0.1399	0.4036
ρ_g	beta	0.500	0.2000	0.986	0.0046	0.9786	0.9936
ρ_i	beta	0.500	0.2000	0.809	0.0355	0.7518	0.8677
ρ_r	beta	0.500	0.2000	0.169	0.1065	0.0211	0.3115
ρ_p	beta	0.500	0.2000	0.965	0.0157	0.9419	0.9894
ρ_w	beta	0.500	0.2000	0.970	0.0125	0.9509	0.9893
μ_p	beta	0.500	0.2000	0.807	0.0531	0.7269	0.8913
μ_w	beta	0.500	0.2000	0.901	0.0312	0.8538	0.9504
φ	norm	4.000	1.5000	5.856	0.9735	4.2712	7.4485
σ_c	norm	1.500	0.3750	1.607	0.1651	1.3421	1.8723
λ	beta	0.700	0.1000	0.710	0.0376	0.6490	0.7724
ξ_w	beta	0.500	0.1000	0.713	0.0548	0.6229	0.8019
σ_l	norm	2.000	0.7500	1.155	0.4633	0.4247	1.8835
ξ_p	beta	0.500	0.1000	0.678	0.0376	0.6161	0.7400
ι_w	beta	0.500	0.1500	0.608	0.1273	0.3987	0.8172
ι_p	beta	0.500	0.1500	0.178	0.0620	0.0765	0.2754
ψ	beta	0.500	0.1500	0.737	0.0787	0.6089	0.8668
ϕ_p	norm	1.250	0.1250	1.613	0.0751	1.4905	1.7379
ϕ_a	unif	0.000	2.3094	-0.023	0.0513	-0.1063	0.0625
ψ_b	unif	0.000	2.3094	0.576	0.1208	0.3771	0.7702
ψ_g	unif	0.000	2.3094	0.067	0.0394	0.0026	0.1323
ψ_{i^s}	unif	0.000	2.3094	0.200	0.1057	0.0275	0.3745
$\psi_{p^{inf}}$	unif	0.000	2.3094	-0.426	0.2123	-0.7636	-0.0771
ψ_{w^s}	unif	0.000	2.3094	-0.234	0.0656	-0.3406	-0.1262
ψ_{η^w}	unif	0.000	2.3094	-0.027	0.1422	-0.2624	0.2048
ψ_{η^p}	unif	0.000	2.3094	-3.055	0.7439	-4.0000	-1.9388
ψ_{y^p}	unif	0.000	2.3094	2.289	1.1832	0.5863	3.9167
ψ_y	unif	0.000	2.3094	-2.408	1.2094	-4.0000	-0.7162
ψ_r	unif	0.000	2.3094	2.840	0.6099	1.8967	3.8972
$\psi_{k^{p,s}}$	unif	0.000	2.3094	-0.923	0.3793	-1.5219	-0.2773
ψ_{k^s}	unif	0.000	2.3094	0.034	0.1919	-0.2774	0.3478
ψ_{c^p}	unif	0.000	2.3094	1.986	1.1566	0.3706	3.9990
ψ_{i^p}	unif	0.000	2.3094	-1.593	0.4862	-2.3668	-0.7811
ψ_c	unif	0.000	2.3094	0.559	0.7923	-0.7227	1.7883
ψ_i	unif	0.000	2.3094	1.094	0.3484	0.5330	1.6482
ψ_π	unif	0.000	2.3094	1.580	0.5500	0.6507	2.4438
ψ_w	unif	0.000	2.3094	-0.396	0.1841	-0.6901	-0.0875

Notes: Estimated model is the Smets and Wouters's [2007] model with a state rule. Sample is 1959Q1-2019Q4.

Table A.15: Parameter Estimates from Smets and Wouters [2007] with State Rule

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
$\bar{\pi}$	gamm	0.625	0.1000	0.609	0.0939	0.4532	0.7600
$100(\beta^{-1}-1)$	gamm	0.250	0.1000	0.160	0.0607	0.0634	0.2550
$\bar{l}$	norm	0.000	2.0000	-1.316	1.4988	-3.7773	1.1507
$\bar{\gamma}$	norm	0.400	0.1000	0.385	0.0198	0.3537	0.4168
ρ_{ga}	norm	0.500	0.2500	0.575	0.0667	0.4654	0.6845
α	norm	0.300	0.0500	0.212	0.0158	0.1859	0.2379
η^a	invg	0.100	2.0000	0.461	0.0242	0.4226	0.5018
η^b	invg	0.100	2.0000	0.214	0.0207	0.1808	0.2486
η^g	invg	0.100	2.0000	0.475	0.0226	0.4379	0.5119
η^i	invg	0.100	2.0000	0.343	0.0284	0.2969	0.3892
η^m	invg	0.100	2.0000	0.231	0.0169	0.2031	0.2583
η^p	invg	0.100	2.0000	0.134	0.0132	0.1120	0.1552
η^w	invg	0.100	2.0000	0.339	0.0208	0.3047	0.3730

Notes: Estimated model is the Smets and Wouters's [2007] model with a state rule. Sample is `1959Q1-2019Q4`.

Table A.16: Unconditional Variance Decomposition of Output

	State Rule	Taylor Rule
TFP	37.76	51.04
Risk Premium	0.50	16.04
Government Spending	2.34	2.43
Investment Technology	7.55	7.95
Monetary Policy	4.45	6.21
Price Markup	19.19	7.44
Wage Markup	29.83	8.70

Notes: The estimated models are the Smets and Wouters's [2007] model with either a Taylor rule or a state rule version. Sample is `1959Q1-2019Q4`.