2023-2024 – Econ 0107 – Macroeconomics I

Lecture 3 : Ricardian Equivalence

(Chapter 10 in LJUNQVIST & SARGENT)

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Changes from version 1.0 are in red

- Key idea: the timing of lump taxes does not matter $\rightsquigarrow$ equivalence of the debt/lump taxes timing
- This is the equivalent in macro of the Modigliani-Miller theorem (the value of a firm is unaffected by how that firm is financed (debt or equity)).
- Formally presented by Barro, JPE, 1974
- It means that the "Keynesian multiplier $\Delta T = \Delta B/P$" ("*a cut in taxes financed by public debt is expansionary*") is fallacious.
- I present the model, then state the result and discuss it.

- N identical households

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

  with all good properties, including $\lim_{c \downarrow 0} u'(c) = +\infty$
- No uncertainty
- The household can invest in a single risk-free asset bearing a fixed gross one-period rate of return $R > 1$: it is a loan to foreigners or to the government.
- 1 unit of b_{t+1} is a piece of paper that is sold R^{-1} units of good in period t and that promises 1 units of good in $t+1$.
- $b > 0$ means that the Hh is net creditor, $b < 0$ net borrower.

- The time t budget constraint (BC) is

$$c_t + R^{-1} b_{t+1} \leq y_t + b_t \tag{2}$$

  with b_0 given.
- Assume that $R\beta = 1$ and that $\{y_t\}_{t=0}^{\infty}$ is a given nonstochastic nonnegative endowment sequence with $\sum_{t=0}^{\infty} R^{-t} y_t < \infty$.
- The extent to which Ricardian Equivalence holds depends on households' access to financial markets. We explore two possibilities.
- The first one is that the household can lend but not borrow: $b_t \geq 0$ for all t.
- The second one is that the household cannot borrow more that it is feasible to repay: $b_t \geq \widetilde{b}_t$ for all t.
- I will refer to this case as the "no financial constraint case".

- This maximum amount $\widetilde{b}_t$ is computed by setting $c_t = 0$ for all t in (2) and solving forward:

$$\widetilde{b}_t = -\sum_{j=0}^{\infty} R^{-j} y_{t+j} \tag{3}$$

  where the following transversality condition have been imposed:

$$\lim_{T\to\infty} R^{-T} b_{T+1} = 0 \tag{4}$$

- This $\widetilde{b}_t$ is referred to as the *natural debt limit* and we will have in the “no financial constraint case” the restriction:

$$b_t \geq \widetilde{b}_t \tag{5}$$

- The the typical intertemporal household problem is here to maximize (1) s.t. (2) and $b_{t+1} \geq \widetilde{b}_t$ (never binding because $\lim_{c \downarrow 0} u'(c) = +\infty$).

$$\max_{b_{t+1}, c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) \right]$$

- The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t = \beta \lambda_{t+1} \\ \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) = 0 \\ \lambda_t \geq 0 \end{cases}$$

- which implies:

$$u'(c_t) = \beta R u'(c_{t+1}) \ \ \forall t \geq 0$$

- The the typical intertemporal household problem is here to maximize (1) s.t. (2) and $b_{t+1} \geq 0$.

$$\max_{b_{t+1}, c_t} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) + \mu_t b_{t+1} \right]$$

- The FOC are

$$\begin{cases} u'(c_t) = \lambda_t \\ R^{-1}\lambda_t - \mu_t = \beta \lambda_{t+1} \\ \lambda_t (y_t + b_t - c_t - R^{-1} b_{t+1}) = 0 \\ \mu_t b_{t+1} = 0 \\ \lambda_t \geq 0 \\ \mu_t \geq 0 \end{cases}$$

- which gives

$$u'(c_t) \geq \beta R u'(c_{t+1}) \quad \forall t \geq 0 \tag{6a}$$

$$u'(c_t) > \beta R u'(c_{t+1}) \text{ implies } \quad b_{t+1} = 0 \tag{6b}$$

- with $\beta R = 1$, this becomes $c_t = c_{t+1}$ when the consumer is not constrained ($b_{t+1} \geq 0$) and $c_{t+1} > c_t = y_t + b_t$ when she is constrained ($b_{t+1} = 0$).

Example 1 : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_h, y_l, y_h, y_l, ...\}$ with $y_h > y_l > 0$, $b_t \geq 0 \; \forall t$.

Example 2a : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, y_h, ...\}$ with $y_h > y_l > 0$ and $b_t \geq \widetilde{b}_t$ is imposed.

Example 2b : $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, y_h, ...\}$ with $y_h > y_l > 0$ and $b_t \geq 0$ is imposed.

Example 3 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$ and $b_t \geq 0$ is imposed.

Example 4 : $b_0 = 0$, $y_t = \lambda^t$ where $1 < \lambda < R$ and $b_t \geq \widetilde{b}_t$ is imposed.

Example 5 : $b_0 = 0$, $y_t = \lambda^t$ where $1 > \lambda$ and $b_t \geq \widetilde{b}_t$ is imposed.

Example 1 $\quad b_0 = 0, \ \{y_t\} = \{y_h, y_\ell, y_h, y_\ell, \dots\}, \ b_t \geq 0 \quad y_h > y_\ell$

$$\text{Max} \sum \beta^t u(c_t) + \lambda\Big(\underbrace{\sum R^{-t} y_t}_{W_0} - \sum R^{-t} c_t\Big)$$

$$W_0 = \sum R^{-t} y_t = (y_h + R^{-1} y_\ell) + R^{-2}(y_h + R^{-1} y_\ell) + R^{-4}(y_h + R^{-1} y_\ell) + \dots$$

$$= (y_h + R^{-1} y_\ell) \times \sum_{t=0}^{\infty} R^{-2t} = \frac{y_h + R^{-1} y_\ell}{1 - R^{-2}}$$

FOC without constraint $b_t \geq 0$: $\quad c_{t+1} = \underset{= c}{c_t} \quad \forall t \quad$ [Assume b_t always > 0]

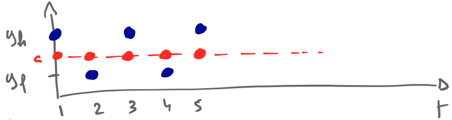
$$\Rightarrow \frac{1}{1-R^{-1}} c = \frac{y_h + R^{-1} y_\ell}{1 - R^{-2}} \Rightarrow c = \frac{1-R^{-1}}{(1-R^{-1})(1+R^{-1})}\left(y_h + R^{-1} y_\ell\right)$$

$$\left(c = (1-R^{-1}) W_0\right)$$

2. An Infinitely Lived-Agent Economy

Solution to Consumption/Saving Decision: Examples

$$c = \frac{y_h + R^{-1} y_l}{1+R^{-1}}$$



in period 1

$$b_1 = y_h - c = \frac{y_h(1+R^{-1})}{1+R^{-1}} - \frac{y_h + R^{-1} y_l}{1+R^{-1}}$$

$$= \frac{R^{-1}(y_h - y_l)}{1+R^{-1}} > 0$$

$\rightarrow$ we check that indeed $b_1 > 0$

in period 2

$$b_2 = y_l + R b_1 - c = y_l + \frac{y_h - y_l}{1+R^{-1}} - \frac{y_h + R^{-1} y_l}{1+R^{-1}}$$

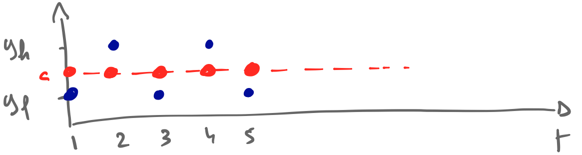
$$= \frac{(1+R^{-1} - 1 + R^{-1})\, y_l + (y_h - y_h)}{1+R^{-1}} = 0$$

Example 2 $\quad \{y_t\} = \{y_l, y_h, y_l, y_h, \dots\} \quad , \; b_t \geq \tilde{b}_t$

$$W_0 = \frac{y_l + R^{-1} y_h}{1+R^{-2}}$$

$$c = (1-R^{-1})\, W_0 = \frac{y_l + R^{-1} y_h}{1+R^{-1}}$$

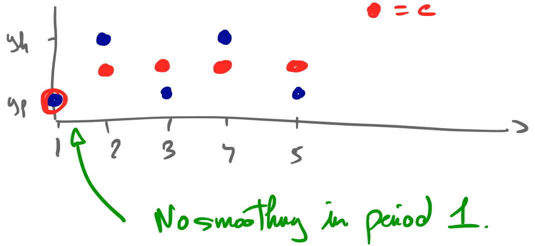
$$b_1 = y_l - c = \frac{R^{-1}(y_l - y_h)}{1+R^{-1}} < 0$$ $\rightsquigarrow$ need to borrow in period 1



Example 2b. $\{y_\ell, y_h, y_\ell, y_h, \dots\}$ and $b_1 \geq 0$

↳ consume $c_1 = y_\ell \Rightarrow b_1 = 0$

↳ in 2, back to example 1



- The Gvt purchases a stream $\{g_t\}_{t=0}^{\infty}$ per household, imposes a stream of lump-sum taxes $\{\tau_t\}_{t=0}^{\infty}$ and is subject to the BC:

$$B_t + g_t = \tau_t + R^{-1} B_{t+1} \tag{8}$$

- B_t is a one-period debt due at t and denominated in period t consumption good. The Gvt is allowed to borrow.
- Ruling out Ponzi schemes ($\lim_{T\to\infty} R^{-T} B_{T+1} = 0$), one gets from solving (8) forward:

$$B_t = \sum_{j=0}^{\infty} R^{-j} \left(\tau_{t+j} - g_{t+j}\right) \tag{9}$$

- The household's BC (2) becomes

$$c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t \tag{10}$$

Solving forward and using the transversality condition:

$$b_t = \sum_{j=0}^{\infty} R^{-j} (c_{t+j} + \tau_{t+j} - y_{t+j}) \tag{11}$$

and the natural debt limit is

$$\widetilde{b}_t = \sum_{j=0}^{\infty} R^{-j} (\tau_{t+j} - y_{t+j}) \tag{12}$$

- Note that the debt limit is greater (I mean more binding) with positive taxes.

Definition 1

Given initial condition (b_0, B_0), an **equilibrium** is a household plan $\{c_t, b_{t+1}\}$ and a government policy $\{g_t, \tau_t, B_{t+1}\}$ such that (a) the government plan satisfies the government BC (8) and (b) given $\{\tau_t\}$, the household plan is optimal.

Proposition 1

Suppose that the natural debt limit prevails. Given initial conditions (b_0, B_0)*, let* $\{\overline{c}_t, \overline{b}_{t+1}\}$ *and* $\{\overline{g}_t, \overline{\tau}_t, \overline{B}_{t+1}\}$ *be an equilibrium. Consider any other tax policy* $\{\widehat{\tau}_t\}$ *satisfying*

$$\sum_{t=0}^{\infty} R^{-t}\widehat{\tau}_t = \sum_{t=0}^{\infty} R^{-t}\overline{\tau}_t \tag{13}$$

Then $\{\overline{c}_t, \widehat{b}_{t+1}\}$ *and* $\{\overline{g}_t, \widehat{\tau}_t, \widehat{B}_{t+1}\}$ *is also an equilibrium where*

$$\widehat{b}_t = \sum_{j=0}^{\infty} R^{-j}\left(\overline{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}\right) \qquad \text{and} \qquad \widehat{B}_t = \sum_{j=0}^{\infty} R^{-j}\left(\widehat{\tau}_{t+j} - \overline{g}_{t+j}\right)$$

In words, the timing of taxes and debt does not matter. What matters is their present value.

3. Government Finance
Proof of Proposition 1

- We need to show *(i)* that the consumption plan $\{\overline{c}_t\}$ and the adjusted borrowing plan $\{\widehat{b}_{t+1}\}$ solve the household's optimum problem and *(ii)* that the altered government tax and borrowing plans continue to satisfy the government's BC.
- $\looparrowright$ *(i)* At time 0, the household face a single intertemporal budget constraint (this is true under the natural debt limit)

$$b_0 = \sum_{t=0}^{\infty} R^{-t}(c_t - y_t) + \sum_{t=0}^{\infty} R^{-t}\tau_t \tag{14}$$

- Therefore, the household's optimal consumption plan does not depend on the timing of taxes, but only on their net present value $\rightsquigarrow$ $\{\overline{c}_t\}$ is still feasible and optimal.
- Having $\{\overline{c}_t\}$, we can construct the sequence of $\{\widehat{b}_{t+1}\}$ by solving the household 's BC (10) forward to obtain (14). To do so, we use a transversality condition $\lim_{T\to\infty} R^{-T}\widehat{b}_{T+1} = 0$. Let's check that it is satisfied if the transversality condition is satisfied for the original borrowing plan:

- In an period $k-1$, solving the BC (10) backwards yields

$$b_k = \sum_{j=1}^{k} R^j \left(y_{k-j} - \tau_{k-j} - c_{k-j}\right) + R^k b_0$$

- which gives

$$\overline{b}_k - \widehat{b}_k = \sum_{j=1}^{k} R^j \left(\widehat{\tau}_{k-j} - \overline{\tau}_{k-j}\right)$$

- which is also, by $\times R^{1-k}$

$$R^{1-k}\left(\overline{b}_k - \widehat{b}_k\right) = R \sum_{t=0}^{k-1} R^{-t} \left(\widehat{\tau}_t - \overline{\tau}_t\right)$$

- The limit of the RHS is zero when $k \to \infty$ because of (13). Then, given that $\{\overline{b}_{t+1}\}$ satisfies the TC, $\{\widehat{b}_{t+1}\}$ does.

↬ *(ii)* Let us show now that the altered government tax and borrowing plans satisfy the government BC. This BC is given by

$$B_0 = \sum_{t=0}^{\infty} R^{-j} \tau_t - \sum_{t=0}^{\infty} R^{-j} g_t$$

From (13), we now that the BC is still satisfied. The sequence of $\widehat{B}_{t+1}$ can then be recovered by solving forward this BC at every period t. $\square$

- The former proposition relies on the fact that the household can undo what the government does by using financial markets.
- This neutrality results does not hold any more in the no-borrowing constraint case.
- Now, a change in the timing of taxes can cause a previously non binding constraint binding.
- We have only a weak form of neutrality

Proposition 2

Consider an initial equilibrium with consumption path $\{\overline{c}\}$ in which $b_{t+1} > 0$ for all $t \geq 0$. Let $\{\overline{\tau}_t\}$ be the tax rate in the initial equilibrium, and let $\{\widehat{\tau}_t\}$ be any other tax rate sequence with same present value and for which

$$\widehat{b}_t = \sum_{j=0}^{\infty} (\overline{c}_{t+j} + \widehat{\tau}_{t+j} - y_{t+j}) \geq 0 \qquad (\star)$$

for all $t \geq 0$. Then $\{\overline{c}_t\}$ is also an equilibrium allocation for the $\{\widehat{\tau}_t\}$ sequence.

- If $(\star)$ is satisfied, then the household can undo the change in the government tax and borrowing plan without hitting the no-borrowing constraint.
- The sequence $\{\overline{c}_t\}$ is therefore feasible, and then, one can proceed as in the preceding proof. $\square$

- Often the Ricardian equivalence results is dismissed as irrealistic because the time horizon of some households is shorter than the government one ("I'll be dead before they start raising taxes to pay back public debt $\rightsquigarrow$ for me, government bonds are net wealth")
- Barro was the first to show that this is not true if generations are linked by bequests.
- The model with borrowing constraints can be reinterpreted in such a way:

4. Linked Generations Interpretation

- Assume that there is a sequence of one-period-lived agents, that value consumption and the utility of its unique offspring:

$$u(c_t) + \beta V(b_{t+1})$$

where b_{t+1} is the amount of bequest that is left to generation $t+1$ and V is the maximized utility of a time $t+1$ agent, recursively defined as

$$\begin{aligned} V(b_t) &= \max_{c_t, b_{t+1}} \{u(c_t) + \beta V(b_{t+1})\} && (16) \\ \text{s.t.} \quad & c_t + R^{-1} b_{t+1} \leq y_t - \tau_t + b_t && (17) \end{aligned}$$

with $b_{t+1} \geq 0$

- This model consumption equilibrium allocations are identical to those of the infinitely-lived one with a no-borrowing constraint. Therefore, the weak version of the Ricardian Equivalence theorem holds.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Intergenerational Redistribution

- Take a tax increase on the current generation that is financed by a tax cut on the next generation
- The current generation would want to reduce bequests –*ie* to borrow against the next generation– in order to maintain same consumption plan
- If this implies negative bequests, then not allowed $\rightsquigarrow$ Ricardian equivalence fails.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Capital Market Imperfections

- Again, I have shown before that only a weak form of Ricardian Equivalence holds if there is a no-borrowing constraint.
- It is also the case if there is a wedge between creditor's interest rate and debtor's one.

5. Reasons for Which the Ricardian Equivalence Theorem Might Not Hold

Distortionary Taxes

- If taxes are distortionary, then their timing affect household's decisions.

6. Is the Ricardian Equivalence (or the lack of) Empirically Relevant?

- Difficult to test directly. The Ricardian argument does not render all fiscal policy irrelevant.
- For example, if the government cut taxes today and households expect this tax cut to be met with future cuts in useless government expenditures instead of future tax increases, households' permanent income increases and so does consumption. $\rightsquigarrow$ but one does not observe directly expectations...
- Some assumptions or implications of the Ricardian Equivalence result can be tested.

- It has been shown that consumers do not smooth consumption as much as Permanent Income theory predicts $\rightsquigarrow$ there are liquidity constraints, financial imperfections,...

- In a consumption equation

$$C = f(\text{income, wealth, fiscal policy, taxes, public debt,...})$$

  the coefficients on taxes and public debt should be zero.
- but a lot of implementation problems (expectations (suppose that the current level of taxes affect expectations about future government expenditures) , simultaneity (shocks to consumption might affect fiscal policy),...)
- Using a Euler equation approach, one might overcome some of those difficulties (expectations), but results are not conclusive.

- A debt-financed reduction in government revenues should not affect interest rates, while it should increase it according to the traditional view (say IS-LM).
- Again, the problem is that it is hard to get rid of the changes in expectations $\rightsquigarrow$ difficult to conclude.