UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

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ASSESSMENT PATTERN	:	**Mid Term**
MODULE NAME	:	**Macroeconomics**
LEVEL:	:	**Postgraduate**
DATE	:	**9th Jan 2023**
TIME	:	**12:00**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

Year
2022/23

Additional material	
Special instructions	
Exam paper word count	

TURN OVER

University College London, 2022-2023
Econ 0107 – Macroeconomics I – Łukasz Rachel & Franck Portier

Midterm Assessment

Answer the four problems. Each problem carries 25% of the total mark.
Students are expected to spend a maximum of 5 hours on the paper.

By submitting this assessment, I pledge my honour that I have not violated UCL's Assessment Regulations which are detailed in the UCL academic manual (chapter 6, section 9 on student academic misconduct procedure), which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.

I – Human capital

This question analyzes an economy in which individuals accumulate human capital subject to idiosyncratic risk. Individuals accumulate human capital in a fashion similar to the Lucas (1988) endogenous growth model. There are two main differences: (a) this human capital accumulation is subject to idiosyncratic risk (which matters because individuals are risk averse), and (b) there is a continuum of individuals subject to this idiosyncratic risk rather than a representative agent.

Individuals have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1$$

where c_t is consumption and the period utility function $u(\cdot)$ is strictly increasing and strictly concave. Individuals are endowed with one unit of time. Human capital h_t accumulates according to

$$h_{t+1} = h_t(1 + g(s_t)),$$

where $s_t \in [0, 1]$ is the fraction of time devoted to human capital accumulation and $g(\cdot)$ is a strictly increasing function satisfying $g(0) = 0$. Given a time allocation s_t consumption is given by

$$c_t = z_t h_t(1 - s_t),$$

where z_t follows a Markov process with transition probabilities $F(z_{t+1}|z_t)$. The idea is that individual productivity fluctuates randomly over time even conditional on a given stock of human capital h_t (some days you're more productive than others; hopefully today is a high-z day for you!).

1 – Write the Bellman equation for the individual optimization problem.

2 – Describe an algorithm for solving the Bellman equation you wrote in question (a). Present the algorithm as numbered steps that one could implement on a computer. As part of this description, please carefully discuss how to convert the problem with continuous state variable(s) into a problem that can be handled on a computer, e.g. via discretization. The algorithm should also feature a stopping criterion.

3 – Derive the first-order equation for the time investment decision s. How do you expect the investment policy function for s to behave as a function of idiosyncratic productivity z? For this question, you can assume $u(c) = c^{1-\sigma}/(1-\sigma), g(s) = \bar{g}s$. No need to solve the problem. It's sufficient to argue verbally using intuition or examples.

4 – What can you say about the existence of a stationary distribution of human capital?

5 – Solve this problem on a computer, assuming that $z = \{0.5, 1, 1.5\}$ with each outcome occurring with probability 1/3 every period, $u(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma = 2$, $g(s) = s$. *Hint 1: you may want to first re-express the problem so that* h_{t+1} *or* $\frac{h_{t+1}}{h_t} \in [1, 2]$ *is the control variable.* Plot the value function and the policy function. Verify your answers to part (3) by also presenting the solution for $\sigma = 1$. *Hint 2: You should submit the code and the results, even if they are unfinished or contain a bug. You are of course welcome to start from an existing piece of code and adjust it appropriately.*

II – An earthquake in a neoclassical growth model

Consider the optimal growth problem in continuous time

$$\max_{[c(t),k(t)]_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} u\left(c\left(t\right)\right) dt$$

$$\text{s.t.} \quad \dot{k}\left(t\right) = f\left(k\left(t\right)\right) - \delta k\left(t\right) - c\left(t\right) \text{ for each } t,$$

$$c\left(t\right), k\left(t\right) \geq 0 \text{ and given } k\left(0\right),$$

and $u\left(c\right)$ and $f\left(k\right)$ strictly increasing, strictly concave, and satisfying the Inada conditions.

1 – Write down the current-value Hamiltonian. What is the interpretation of the costate variable in this setting? (use the notation μ for the costate variable)

2 – Write down the optimality conditions.

3 – Derive the Euler equation.

4 – In lectures on several occasions we have discussed a partial equilibrium consumption saving problem. The Euler equation in that case was $\frac{\dot{c}}{c} = \mathcal{E}(c)(r - \rho)$. We derived the conditions for consumption path to be either flat, increasing or decreasing forever. Does similar analysis apply to the model in this question? Why / why not?

5 – For this subquestion only, consider the decentralised equilibrium of this economy. What is the long-run elasticity of capital supply with respect to the interest rate in this economy? How does this elasticity compare to the one usually found in Bewley-Hugget-Aiyagari type models?

6 – Assume $u(c) = \frac{1-e^{-ac}}{a}$ with $a > 0$. Draw a phase diagram in the k, c space. Illustrate the dynamics and draw the saddle path.

7 – Calculate the steady state value of capital and consumption, assuming that the production function is $f(k) = k^{\alpha}$. How do the steady state values depend on a? Provide some intuition.

8 – Suppose the economy is in steady state. Then an earthquake wipes out around a third of the capital stock in the economy. Mark the position of the economy right after the earthquake on the phase diagram you have drawn. Explain briefly how the post-earthquake consumption is determined.

9 – Suppose the economy is in steady state. Then there is a sudden increase in a. Interpret this shock (in one sentence). Trace out the dynamic path following the shock on the phase diagram. In a separate figure, plot the time paths of k and c. Explain which variables jump (if any) and why.

10 – Now assume that, starting from the steady state, the increase in a came together with an earthquake that wiped out around a third of the capital stock in the economy. Trace out the dynamic path following the shock on the phase diagram. On a separate figure, plot the time paths of k and c. Explain which variables jump (if any) and why.

11 – Assume that the increase in a came together with an earthquake that wiped out around a third of the capital stock in the economy, but the increase is temporary: a will remain high for T years and will then revert back to its pre-earthquake level. Trace out the dynamic path following the shock on the phase diagram. On a separate figure, plot the time paths of k and c. Explain the logic for the paths you have drawn and give some economic intuition.

12 – Finally, draw a phase diagram in the k, μ space. Trace out the effect of a shock to a only, as in question **7** – above. Interpret your results.

III – The yield curve

Consider a deterministic infinite horizon endowment economy with time-0 trading. There are N agents indexed by $i = 1, \dots, N$ and endowments are $y^i = \{y_t^i\}_{t=0}^{\infty}$. Preferences are

$$U^i(c^i) = \sum_{t=0}^{\infty} \beta^t \log c_t^i.$$

We assume that agents trade assets in period 0. Asset "t" has a price q_t^0 and pays one unit of good in period t.

Let's first assume that $y_t^i = y \; \forall i$ and $\forall t$.

1 – Write the maximisation problem of agent i, denoting μ^i the Lagrange multiplier of the intertemporal budget constraint. Derive the first order condition with respect to c_t^i.

2 – Define a competitive equilibrium for this economy. Solve for prices and quantities (you can use the normalisation $q_0^0 = 1$).

3 – Define $r_{t,t+1}$ by

$$\frac{q_{t+1}^0}{q_t^0} = \frac{1}{1 + r_{t,t+1}} \approx e^{-r_{t,t+1}}$$

and $r_{0,t}$ by

$$q_t = e^{-r_{0,1}} e^{-r_{1,2}} \cdots e^{-r_{t-1,t}} = e^{-t r_{0,t}}$$

with $r_{0,t} = \frac{1}{t}(r_{0,1} + r_{1,2} + \cdots + r_{t-1,t})$. Compute $r_{0,t}$ $\forall t$ and plot the yield curve –i.e $r_{0,t}$ as a function of t.

4 – Assume now that $y_t^i = \gamma^t y$ with $\gamma > 1$. Compute $r_{0,t}$ $\forall t$ and plot the yield curve.

5 – Assume now that $y_0^i = y \; \forall i$, $y_t^i = \gamma_1^t y_{t-1}^i$ for $t \leq T$ and $y_t^i = \gamma_2^t y_{t-1}^i$ for $t > T$, with $\gamma_1 > 1$ and $\gamma_2 > 1$. Compute $r_{0,t}$ $\forall t$ and plot the yield curve for the two cases $\gamma_1 > \gamma_2$ and $\gamma_1 < \gamma_2$. Discuss.

6 – Let's assume now that there are two agents in the economy, indexed by 1 and 2. They behave in a competitive way. Endowments of agent 1 is $y^1 = \{y + \varepsilon, y - \varepsilon, y + \varepsilon, y - \varepsilon, \dots\}$ and endowment of agent 2 is $y^1 = \{y - \varepsilon, y + \varepsilon, y - \varepsilon, y + \varepsilon, \dots\}$, with $|\varepsilon| < y$. Prove that in a competitive equilibrium $\frac{c_t^1}{c_t^2}$ is a constant that depends of $\frac{\mu^1}{\mu^2}$. What does that mean in terms of risk sharing.

7 – Compute $\frac{\mu^1}{\mu^2}$ and show that $\frac{\mu^1}{\mu^2} < 1$ when $\varepsilon > 0$. Why?

IV – Bubbly asset in an Overlapping Generations model

Consider an economy in which overlapping generations of agents live for two periods (young and old). The size of each generation is normalised to one. Preferences are $u(c_t^y) + u(c_{t+1}^o)$. Endowment is ω^y when young and ω^o when old. There exist an asset in quantity A. Each unit of the asset pays dividend $d \geq 0$ every period and has non negative price p_t. In period 0, the old generation holds the A units of the asset. Young agents of period t can save by buying a_{t+1} units of the asset at price p_t or by subscribing a private bond in quantity b_{t+1}. The price of the bond is one, and it pays $(1+r_{t+1})$ in period $t+1$. The bond is in zero net supply. We assume that the natural debt limit holds. When a young buys one unit of asset, she receives the dividend when old, and then resells the asset.

1 – Write the budget constraints of generation t when young and when old. Use them to prove that in equilibrium, one must have

$$p_t = \frac{p_{t+1} + d}{1 + r_{t+1}}.$$

2 – Derive the Euler equation of a generation t individual

3 – Define competitive equilibrium in that economy.

4 – We now restrict to steady states. Assume that $d > 0$. Show that one cannot have $r \leq 0$ at the steady state. Compute the unique steady state in that case.

5 – Assume now that $d = 0$. Show that autarky is always a steady state. Show that when $\omega^y > \omega^o$, there is another steady state in which $p > 0$. Compute that steady state. Why is that a bubbly steady state?